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Kinetic Simulations of the Formation and Stability of the Field-Reversed Configuration

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Abstract — The Field-Reversed Configuration (FRC) is an elongated compact toroidal plasma confined primarily by poloidal fields. In the FRC the external field is reversed on axis by the diamagnetic current carried by thermal plasma particles. A 3-D, hybrid, Particle-in-Cell (PIC) code (zero-inertia fluid electrons and kinetic ions), FLAME [1] previously used to study ion rings is applied to investigate FRC formation and tilt instability. Axisymmetric FRC equilibria are obtained by simulating the standard experimental reversed theta-pinch technique. These are used to study the nonlinear tilt mode in the "kinetic" and "fluid-like" cases characterized by "small" (~ 3) and "large" (~ 12) ratios of the characteristic radial plasma size to the mean ion gyro-radius, respectively. The formation simulations have revealed presence of a substantial toroidal (azimuthal) magnetic field inside the separatrix, generated due to the stretching of the poloidal field by a sheared toroidal electron flow. This is shown to be an important tilt-stabilizing effect in both cases. On the other hand, the tilt mode stabilization by finite Larmor radius effects has been found relatively

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insignificant for the chosen equilibria.

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I Introduction

The FRC as an alternate magnetic confinement concept [2] is the only system under active experimental investigation today where a substantial fraction of the ions have large orbits. This is a result of the existence of a magnetic null point and a separatrix in the vicinity of which ion Larmor trajectories are finite. Simple geometry and naturally high beta make the FRC attractive for fusion applications. A conventional measure for characterizing FRCs as "kinetic" and "fluid-like" is the parameter s, which is the approximate number of ion gyro-radii between the o-point and the separatrix, $s \simeq s^* = (r_s - r_o)\omega_{pi}/c$ ($\beta \sim 1$, ω_{pi} is the peak ion plasma frequency, r_s, r_o are the separatrix and magnetic null radii, respectively). Although the FRC concept is not new, it has not yet been experimentally investigated in reactor relevant regimes, $s \geq 10$. Most theoretical studies of FRC equilibrium have been carried out in past years with the help of ideal-MHD codes [3],[4]. Their results clearly predict that the typical fluid-like FRC should become unstable due to the global and highly destructive tilt mode, developing rapidly with the typical growth rate $\gamma_{MHD} \sim 2v_A/l_s$ (l_s is the separatrix axial length, v_A is the Alfvén velocity calculated with respect to the peak density at the magnetic null point and the axial magnetic field at the separatrix midplane point) [3]. This instability was first theoretically predicted for a prolate spheromak [5] and is physically related to the flipping of a magnetic dipole placed in a reversed external magnetic field. Since the tilt is internal to the FRC separatrix, it is hard to stabilize by external means.

Tilt-like activity and related FRC degraded confinement were reported in Los Alamos experiments [6]. This was supported by results from fluid simulations. In many observations, however, the FRC proved to be more stable. The best confinement occured for $s \sim 1.5-2$, $E \sim 4$ ($E = l_s/2r_s$ is the FRC elongation) and was attributed to a combination of collisional and kinetic effects. In other experiments carried out at University of Washington [7] no correlation was observed between tilt-like signals and gross FRC stability. The high-s ($s \sim 6-8$) FRCs were also reported to persist for more than several tilt growth times, $\tau_{MHD} \sim 1/\gamma_{MHD}$ [8]. In order to explain these "anomalous" stability properties, Hall-fluid [9] and gyroviscous calculations [10] have been applied to study the tilt instability. Their results suggested tilt stabilization for $s/E \leq 1/4$. Most recently, the "hollow" current profiles and "racetrack" (as opposed to "elliptic") separatrix shapes were theoretically and numerically found to be ideally stable to tilting [11],[12]. However, despite these recent advances, ideal MHD theory still falls short of providing a robust mechanism that would explain experimentally observed stabilization or at least substantial growth rate reduction. Indeed, searching for stable regimes in a large space of FRC parameters is a nontrivial task.

This work focuses on kinetic FRC modeling using a hybrid, PIC computational model based on a fluid-electron, particle ion representation. This common paradigm has been used as a basis to investigate the FRC stability to the tilt mode in two previous studies [13],[14]. In the first approach [13] kinetic growth rates of the tilt instability were obtained from a Vlasov-fluid dispersion functional separated into fluid and kinetic portions. The latter was evaluated by a PIC code that followed equilibrium ion orbits and used the relevant MHD eigenmode as a trial function in the dispersion functional. This resulted in a computation of linear growth rates, which were shown to be greatly decreased (compared to the MHD prediction) for $s \leq 2$ and approached the MHD value for $s \sim 15$. In the second approach [14] the tilt mode was simulated directly with a hybrid code, QN3D [14]. Results of these simulations were reported to be generally consistent with the linear (kinetic) predictions. Particularly, it was found that the tilt growth rate was significantly reduced in in the $s \sim 2$ case compared to the fluid-like FRC ($s \sim 12$). In these runs the tilt mode ($m = 1$) rapidly grew out of the numerical noise (the number of particles was rather small because of CPU limitations), while other ($m > 1$) harmonics were also present in the system. It was subsequently suggested that the tilt mode can be stabilized by injecting an energetic ion beam into the FRC [15]. It should be noted that in these studies the FRC equilibria were generated by solving the axisymmetric Grad-Shafranov equation (with zero toroidal field) derived either in ideal MHD or two-fluid approximation.

It is the purpose of this work to investigate the tilt stability properties of the "kinetic" and "fluid-like" FRCs formed by magnetic implosion ("θ-pinch"). The paper is organized as follows. Sec. 2 briefly describes the numerical model. The axisymmetric FRC equilibria are obtained in Sec. 3. The stability of these equilibria to the tilt mode is investigated in Sec. 4. Finally, Sec. 5 summarizes the results of this work.

II Numerical Model

The FRC formation and equilibrium are studied with a hybrid, PIC code, FLAME developed specifically to model global FRC/Ion Ring systems on massively parallel computing facilities [1]. FLAME follows full-orbit ion motion and provides a method for nonlinear stability analysis that does not break down near the field null or the separatrix. The electrons are modeled as a massless fluid collisionally coupled with the ions and cold neutrals. The code assumes plasma quasineutrality and solves the Maxwell equations without displacement current:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}(\mathbf{j}_e + \mathbf{j}_i), \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}, \tag{2}$$

$$\mathbf{E} = \eta_e \mathbf{j}_e + \eta_{ei}\mathbf{j}_i + \frac{\mathbf{j}_e \times \mathbf{B}}{en_e c} - \frac{\nabla p_e}{en_e}, \quad \eta_e = \eta_{ei} + \eta_{en} \tag{3}$$

$$\frac{1}{\gamma - 1}\frac{\partial p_e}{\partial t} = -\frac{1}{\gamma - 1}\nabla(p_e \mathbf{v}_e) - p_e \nabla \mathbf{v}_e + Q_e, \tag{4}$$

where η_{ei}, η_{en} are the electron resistivities due to collisions with the ions and neutrals, respectively. Q_e is the electron heat source and γ is the adiabatic constant.

The ions are represented by macro-particles whose velocities are advanced by the nonrelativistic collisional Newton-Lorentz equations:

$$\frac{d\mathbf{v}_i}{dt} = \frac{q_i}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nu_s \mathbf{v}_i + \left(\frac{d\mathbf{v}_i}{dt}\right)_{ii} + \left(\frac{d\mathbf{v}_i}{dt}\right)_{ie} \tag{5}$$

where ν_s is the ion slowing-down collision frequency. The last two terms in (5) represent ion-electron and ion-ion collisions, respectively. They are calculated from a grid-based collision model [16]. FLAME uses the cylindrical (r, θ, z) coordinates and exploits the periodic nature of the azimuthal (θ) direction by representing all grid quantities as finite Fourier series. This makes it possible to follow only selected azimuthal modes that are of particular interest to us in this study. This numerical algorithm is described in detail elsewhere [1].

III FRC Formation

Most FRCs are formed on fast time-scales ($\sim 10\ \mu sec$) by the field-reversed θ-pinch (FRTP) technique [17], [18]. In this method the discharge tube is filled with a neutral gas and a bias magnetic field is applied. Following the gas ionization the bias field becomes frozen into a conducting plasma. The coil current is then quickly reversed, producing an inductive electric field that causes radial plasma implosion and magnetic reconnection. The magnetic tension forces the FRC to contract axially until equilibrium is reached. This process has been previously simulated in 2-D both in the MHD [19] and kinetic [20] regimes. Both (axisymmetric) simulations used the magnetic flux (ψ) representation and postulated temporal flux evolution at the wall. In this study, FRC formation is also assumed to be axisymmetric, but in contrast to the above works, the wall boundary condition postulates the rate at which the magnetic field *strength* is ramped up in time. In the experiments, magnetic reconnection near the ends occurs mostly due to an initial axial inhomogeneity of the external magnetic field produced by the additional end coils. For simplicity, the closing of magnetic field lines in this simulation is achieved by increasing the plasma (electron) resistivity towards the device ends during the initial stage of magnetic implosion (physically, this can be caused by the presence of neutral gas in the end regions). The purpose of these simulations is to produce FRCs for the stability studies. Therefore, only the most essential physics is retained in the present FRC model. This is done by assuming zero electron pressure in Eqs. (3)-(4), neglecting ion current in Eq. (3) and dropping ion-electron and slowing-down collision terms in Eq. (5). A self-consistent simulation of FRTP formation (including finite electron pressure and realistic resistive effects) requires a detailed energy transport model and will be the subject of a separate work.

The simulation scenario is as follows. We focus on two cases, corresponding to the "kinetic" and "fluid-like" FRCs with $s^* = 3$ and $s^* = 12$, respectively. These runs produce equilibria with similar sizes and ion temperatures (Table 1). The axial magnetic field is ramped up at the wall in time as shown in Fig. 1. The radial implosion of the magnetic field and FRC formation occur on a resistive time scale, $\tau_\eta = 4\pi R^2/\eta_e c^2$ (R is the characteristic radial plasma size), which is much longer than the Alfven radial transit time, $\tau_A = R/v_A$. Experiments consistently produce anomalously high estimates of the plasma resistivity during the dynamic theta-pinch implosion stage. It is thought to be caused by a number of micro-instabilities and usually calculated from empirical laws [19]. For simplicity, we assume constant η_e (except near the FRC ends) with typical magnetic Reynolds numbers, $R_M = \tau_\eta/\tau_A \sim 50$ (Table 1).

Experimental FRC equilibria are presumed (see later) to be produced with little toroidal magnetic field. As a result, previous theoretical studies have commonly assumed $B_\theta = 0$ in equilibrium. On the other hand, kinetic simulations of FRC formation [20] revealed spontaneous generation of a large toroidal field. In this work we consider both scenarios as follows. For each of the above cases, a "reduced" simulation is conducted first by zeroing out B_θ in Eq. (1)-(4). Second, a "full" simulation is carried out. These runs produce axisymmetric kinetic and fluid-like FRC equilibria with and without toroidal magnetic field.

A $B_\theta = 0$

The plasma location as a function of time during the θ-pinch FRC formation in the absence of toroidal magnetic field is displayed in Fig. 2(a) ($s^* = 3$) and Fig. 2(b) ($s^* = 12$). Contour plots of the poloidal flux function, ψ for these cases are shown in

Fig. 3(a) and Fig. 3(b), respectively. In both cases the initial plasma density profiles are assumed to fall exponentially in the axial direction. This approximates the loss of initial plasma out of the ends of the device. The initial radial profile is irrelevant in the kinetic case as it is rapidly modified by the imploding magnetic field and ensuing particle drifts. Therefore, it was assumed close to uniform. On the contrary, smaller radial particle drifts cause the magnetic implosion of the "fluid" plasma to take place predominantly in a thin layer in the vicinity of the plasma-vacuum interface. As a result, the radial current density profile of the fluid-like FRC is narrower (Fig. 4(b)) than that of the kinetic FRC (Fig. 4(a)). Thus, only the very outer part of the plasma density profile undergoes substantial change during the implosion. The rest of the plasma (closer to the axis) carries less current and the description of its radial profile becomes irrelevant as well. To make the fluid equilibrium ($s^* = 12$) look similar to the self-similar kinetic one, the initial radial profile was chosen to peak near the plasma-vacuum boundary. It should also be noted that in this case the axial current distribution is highly peaked towards the ends of the FRC (Fig. 4(b)). As a result, the separatrix of the fluid-like FRC has a racetrack shape (Fig. 3(b)) but the kinetic equilibrium is much better approximated by an elliptical profile (Fig. 3(a)). Therefore, while two equilibria have almost identical elongation (see Table 1), they have different shape indices. The shape index is defined as the elongation of the separatrix end cap and is an important geometric stability factor [12] because it characterizes the proximity of bad curvature regions (near the FRC ends) to the symmetry plane.

B $B_\theta \neq 0$

In these runs toroidal field is retained in the Maxwell equations. Time evolutions of the poloidal and toroidal components of the magnetic field for both cases $s^* = 3, 12$

are shown in Fig. 5 and Fig. 6, respectively. The most striking feature of these sumulations is spontaneous generation of a large-scale toroidal magnetic field far exceeding in magnitude fields observed in single-fluid FRC formation studies. This surprising generation of a substantial toroidal field near magnetic field reconnection points was previously reported and thoroughly investigated in hybrid-PIC and two-fluid simulations [20]. They revealed that the toroidal magnetic field arises mostly from retention of the Hall term which is automatically present in the two-component hybrid model. As a result, poloidal components of the magnetic field frozen into a sheared electron flow are stretched in the toroidal direction. The initial appearance of small-scale magnetic islands along the field null in the $s^* = 3$ case (Fig. 5(a)) is caused by ion kinetic motion associated with small B_θ-cells generated by streaming particles reflected from the impinging magnetic field [21],[20]. It should be noted that the symmetric nature of FRC formation provides for equal and opposite values of toroidal field on the opposite sides of the axial midplane. Therefore, the net toroidal flux in the FRC remains negligible (strictly zero in the absence of PIC collisions) at all times. The toroidal field is localized near the ends of the FRC plasma (Fig. 6). Zoomed-in contour maps of the poloidal and toroidal fields are shown in Fig. 7. Obviously, the poloidal flux lines in the central region of the "fluid-like" FRC form a doublet-like structure. In the kinetic case, this structure is smeared out by ion finite-size Larmor motion. The maximum amplitude of the toroidal self-field reaches approximately 30% of the axial field component calculated at the separatrix midplane point (Fig. 8).

IV FRC Stability

It is well known that stability of toroidal plasmas with respect to global ideal modes is achieved by applying a strong toroidal magnetic field produced by external currents. The most dangerous mode leading to the disruption of confinement in such devices (tokamaks, reversed-field pinches) is the kink mode. The conventional FRC is presumed to have little toroidal field and, therefore, should be easily unstable to the lowest order ($m = 1$) kink mode commonly referred to as tilting. Recently, a theoretical analysis of a special range of ideally MHD stable equilibria has been reported, where stability properties are promoted by hollow current profile and racetrack separatrix shape [12]. These equilibria, however, cannot be achieved in a typical θ-pinch system without additional coil design. Therefore, the "anomalous" global stability of routinely produced θ-pinch formed FRCs is usually attributed to nonideal effects such as gyroviscosity and pitch-angle collisions (in cold plasmas). The absence of toroidal field in the FRC is usually automatically identified with the absence of external toroidal field coils, which implies a much simpler magnetic geometry (no on-axis structures) and high-β confinement. On the other hand, the global MHD stability characteristics of systems with a current carried through the center conductor have been found to improve significantly due to the effective stabilization of kink modes by toroidal field pressure [22].

As shown above, the FRC formation simulations indicate automatic generation of a large toroidal field component by poloidal plasma currents during θ-pinch implosion. In order to investigate the role of self-generated B_θ on the FRC tilt stability, the previusly obtained axisymmetric equilibria (with and without toroidal field) are chosen now as the initial conditions for the 3-D simulations. The conversion to three dimensions is done by distributing the desired inventory of macro-particles and grid-defined data uniformly in the θ-dimension (each particle being replaced by a set of equidistant particles with a random toroidal phase) and imposing an internal tilt-like ($m = 1$) perturbation (as a function of the magnetic flux, vanishing at the separatrix) to the axial particle velocities. Alternatively, each particle could be distributed randomly in the θ-dimension so that the tilting mode would grow out of the initial noise as in Ref. [14]. However, in this study the present approach is deemed much "cleaner" and more appropriate for the purpose of studying the tilt instability. Also, for these very reasons, only the lowest modes ($m = 0, 1$) are retained in Eqs. (1)-(4). In order to save CPU time in three dimensions, the velocity perturbation amplitude was chosen to be fairly large $\sim 0.05 v_A$ and the ion-ion collision term in the ion equation of motion Eq. (5) was dropped (its stabilizing role was found to be insignificant for the given FRC parameters).

A $B_\theta = 0$

Contrary to the previous kinetic studies [14] our simulations do not not seem to support the hypothesis of the tilt mode stabilization by the kinetic (small-s^*) effects. Fig. 9 and Fig. 10 demonstrate a temporal evolution of the "generalized" flux function ψ (contours of constant ψ in three dimensions do not coincide with magnetic flux surfaces but provide a good qualitative indication of the tilting) and plasma density, respectively. These plots and Fig. 11 (a snapshot of a tilted particle distribution) show very clearly that both equilibria ($s^* = 3, 12$) become visibly tilted after several characteristic growth times, τ_{MHD} with initial growth rates (calculated by computing the magnetic energy of $m = 1$ mode) being close to the MHD growth rate, γ_{MHD}. A somewhat surprising finding is that the rate of axisymmetry loss in the kinetic case

(Figs. 9(a),10(a)) is more pronounced compared to the fluid one (Figs. 9(b),10(b)). This apparent discrepancy can be explained in the following manner. As mentioned above, the shape of the fluid-like FRC approches that of a racetrack as opposed to the elliptical shape of the kinetic configuration. The aforementioned theoretical analysis conducted within the framework of ideal MHD theory [12] shows that the crowding of internal field lines closer toward the FRC ends (the "racetrack" case) results in weakening the destabilizing curvature term and strengthening the stabilizing magnetic compressibility term in the energy principle equation. Thus, in our simulations the racetrack-shape index proves to contribute more to the FRC stability than the kinetic effects. However, these factors are too weak to considerably improve the long-term stability of such configurations. This result has recently been confirmed in independent kinetic studies of FRC global stability [23].

B $B_\theta \neq 0$

Finally, we investigate the tilt stability of the FRCs obtained in the "full" formation runs. These are characterized by spontaneous generation of a toroidal magnetic field, B_θ of the same order of magnitude as the imploding polidal fields. The qualitative results are quite clear. In both cases ($s^* = 3, 12$) finite-B_θ configurations show only slight tilting if any at all, as shown in Fig. 12. This is further confirmed by a dramatic difference observed between the time histories of the $m = 1$ mode magnetic energy calculated in the reduced ($B_\theta = 0$) and full ($B_\theta \neq 0$) simulations (Fig. 13). Indeed, the FRCs with a finite self-B_θ retain their axisymmetric shape for many growth periods (even though they are further contracting in the axial direction).

The stabilizing influence of the self-generated toroidal field is two-fold. First, it acts to increase the total magnetic pressure near the plasma ends in the region where the poloidal field is negligible and the field lines have bad curvature. As a result, tilt growth is significantly reduced. In terms of the energy principle equation this corresponds to a greatly increased stabilizing influence of the magnetic compressibility term. Earlier, an externally applied toroidal field was proven to stabilize ultra low aspect ratio tokamaks against the tilt/shift instability [22]. Second, the self-generated toroidal field is found to be closely related to the large-scale poloidal ion flows with velocities of the order of the Alfvén speed. These flows can effectively disrupt the tilt mode structure.

V Summary and Conclusion

It has long been speculated that the global tilt stability of experimentally observed FRCs is mostly due to some combination of kinetic and geometric effects. This work, however, provides a new alternative explanation. Using a hybrid simulation model (fluid elecrons and kinetic ions), the axisymmetric formation of kinetic amd fluid-like FRCs has been studied in two cases: first, by forcing $B_\theta = 0$ in the Maxwell equations and second, by conducting a full simulation ($B_\theta \neq 0$). Contrary to the results of ideal (one-fluid) MHD simulations the full runs display a self-generated quasisteady toroidal field of considerable magnitude. Its establishment in the fluid-like FRC is accompanied by a splitting of the poloidal flux surfaces into a doublet-like structure. Earlier, this effect was demonstrated and traced to the retention of the Hall term in the hybrid models [20]. Interestingly, the equilibria obtained in the formation runs with $B_\theta = 0$ have been found to be unstable to tilting, regardless of the value of s^* or shape index. In contrast to this, the FRCs with finite self-generated toroidal self-fields are shown to be grossly stable.

It is commonly assumed that the FRCs produced by the θ-pinch technique have little toroidal field inside them. The previous theoretical stability analyses have routinely neglected equilibrium toroidal field. As a result of this approach, both the MHD and recent kinetic simulations predicted rapid FRC termination due to the internal tilt [23]. Remarkably, the results obtained in this work indicate the importance of considering a different class of the FRC equilibria characterized by the existence of large-scale self-generated toroidal fields and poloidal flows.

Unfortunately, it is difficult to measure such fields experimentally, for any internal magnetic probing can easily terminate the FRC faster than the tilting itself. Nevertheless, the spontaneous generation of a toroidal field was indeed recorded in rotamak experiments, where a rotating magnetic field was used to produce the steady-state FRC [24]. The probe measurements clearly confirmed the existence of quasisteady toroidal fields. It was observed that the low-frequency component of the toroidal flux in each half of the the discharge vessel was of opposite sign. Moreover, the relative directions and magnitudes of the poloidal and toroidal fields obtained in the rotamak measurements are very similar to those found in this theoretical study. On the other hand, the existence of substantial steady-state toroidal fields has never been explicitly confirmed in the θ-pinch experiments since the magnetic probes in these experiments typically measure B_θ outside the FRC separatrix or/and relatively near the midplane of the device. Understandably, probes in these positions cannot be exposed to large toroidal fields.

Thus, in order to be able to study the toroidal self-field effects in the FRC, the experimental devices should be properly equipped with toroidal flux loops positioned inside the separatrix in different (r, z) planes around the toroidal direction. It is also important to focus the main thrust of future theoretical efforts on investigating new "unorthodox" FRC equilibria that might reflect experimental conditions more adequately. In summary, it is very encouraging that this approach has already resulted in identifying grossly stable FRC equilibria consistent with available experimental evidence.

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Table 1 (Summary of FRC Runs)

Run	$s^* = 3$	$s^* = 12$
Wall Radius	15 cm	15 cm
Reynolds Number	50	50
Background $\mathbf{B_z}$	1.25 kG	5 kG
Self-generated $\mathbf{B}_\theta$	0.2 kG	1.5 kG
Peak Density	5×10^{14} cm^{-3}	0.8×10^{16} cm^{-3}
Ion Temperature	100 eV	100 eV
Separatrix Length	42 cm	44 cm
Separatrix Radius	11.5 cm	11.5 cm
O-Point Radius	8.5 cm	8.5 cm
Elongation	2	2
Alfven Velocity	1.2×10^7 cm/s	1.2×10^7 cm/s
MHD Growth Time	1.65 μs	1.65 μs
Grid Size $(\mathbf{z}, \mathbf{r}, \theta)$	$80 \times 45 \times 16$	$80 \times 45 \times 8$
$\mathbf{N_{PIC}}$ (3-D)	$2,400,000$	$1,200,000$

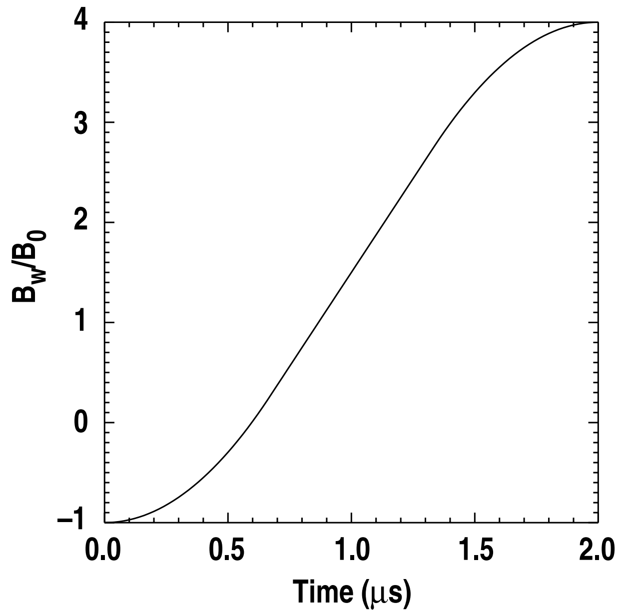

FIG. 1. Time history of axial magnetic field at the cylindric wall during θ-pinch compression.

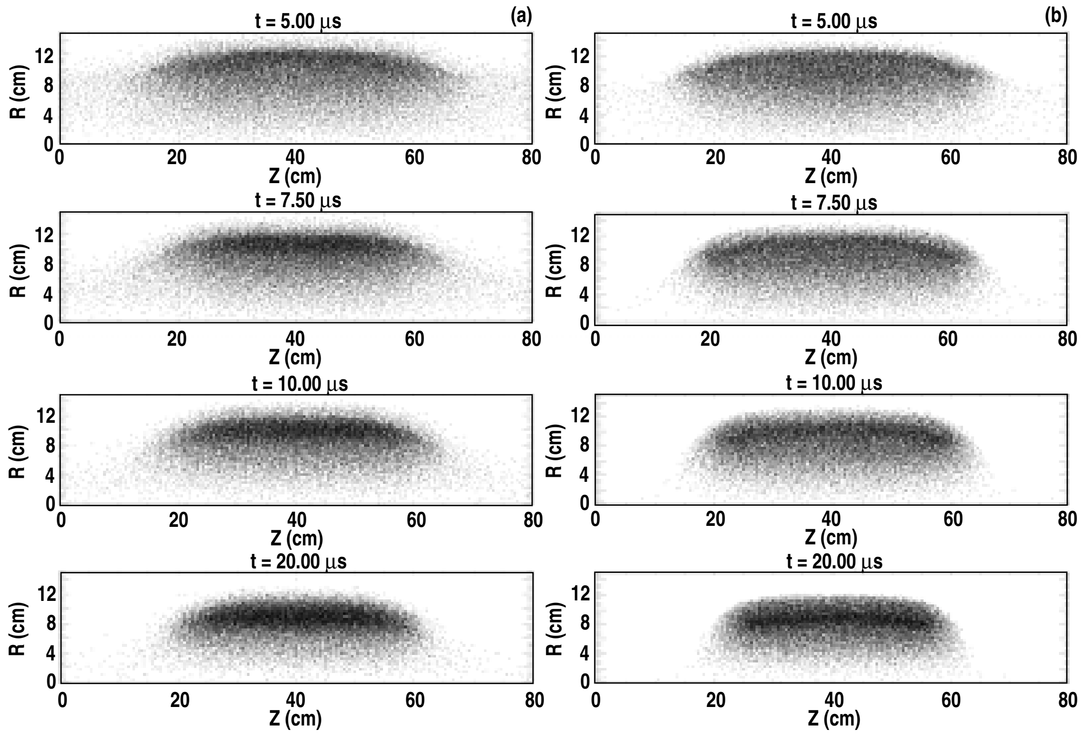

FIG 2. Time evolution of poloidal particle distribution (axisymmetric formation, $B_\theta = 0$): (a) $s^* = 3$; (b) $s^* = 12$.

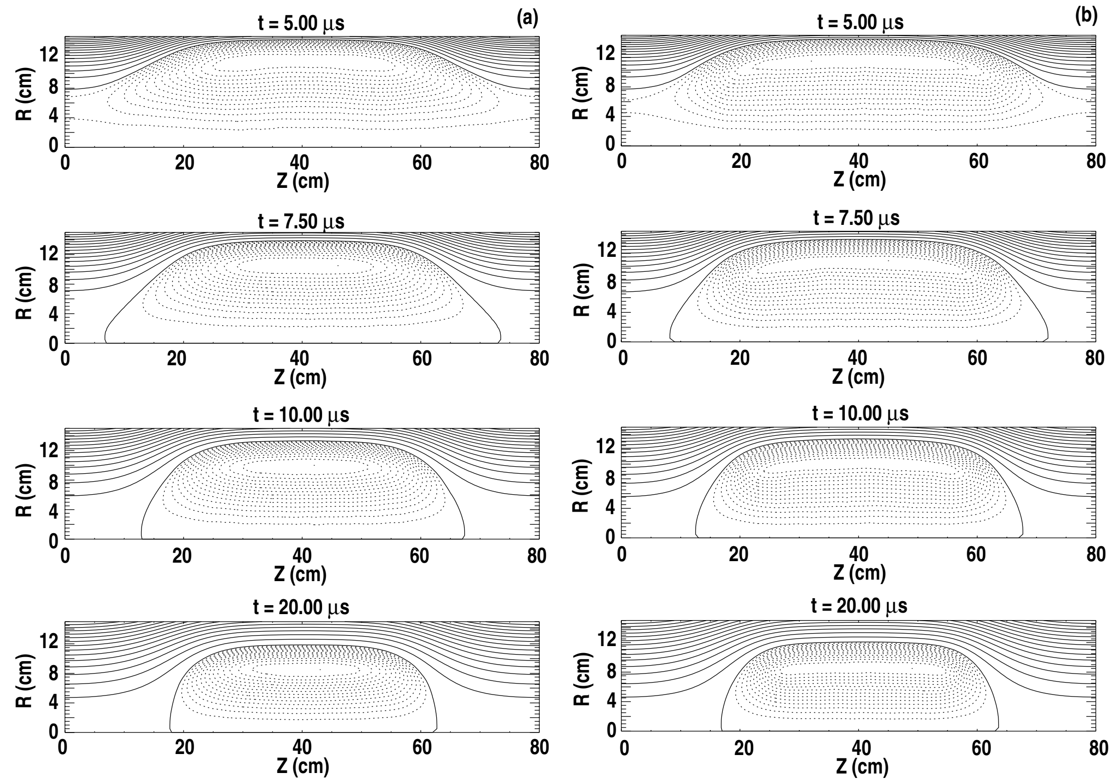

FIG. 3. Time evolution of poloidal flux function ψ (axisymmetric formation, $B_\theta = 0$): (a) $s^* = 3$; (b) $s^* = 12$.

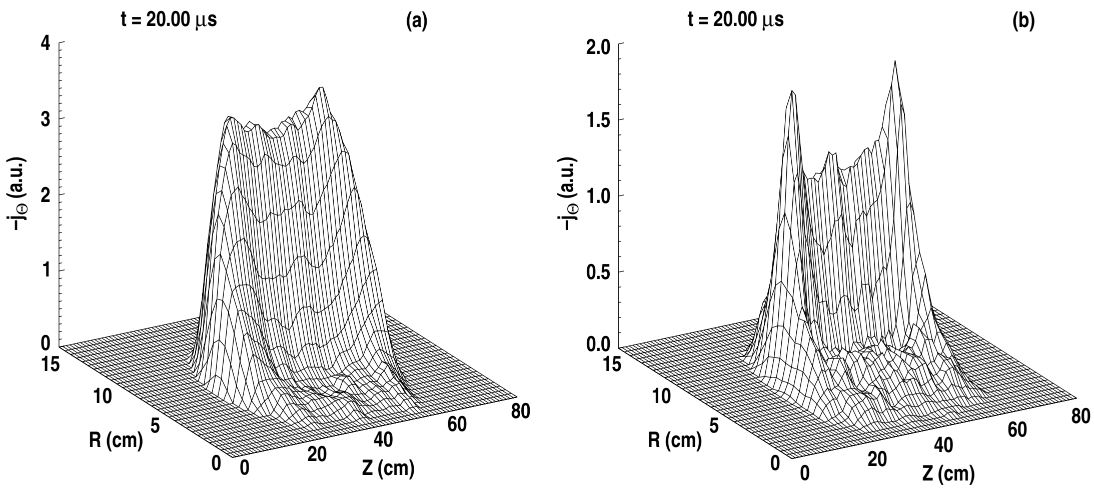

FIG 4. Comparison of radial profiles of toroidal current density ($B_\theta = 0$): (a) $s^* = 3$; (b) $s^* = 12$.

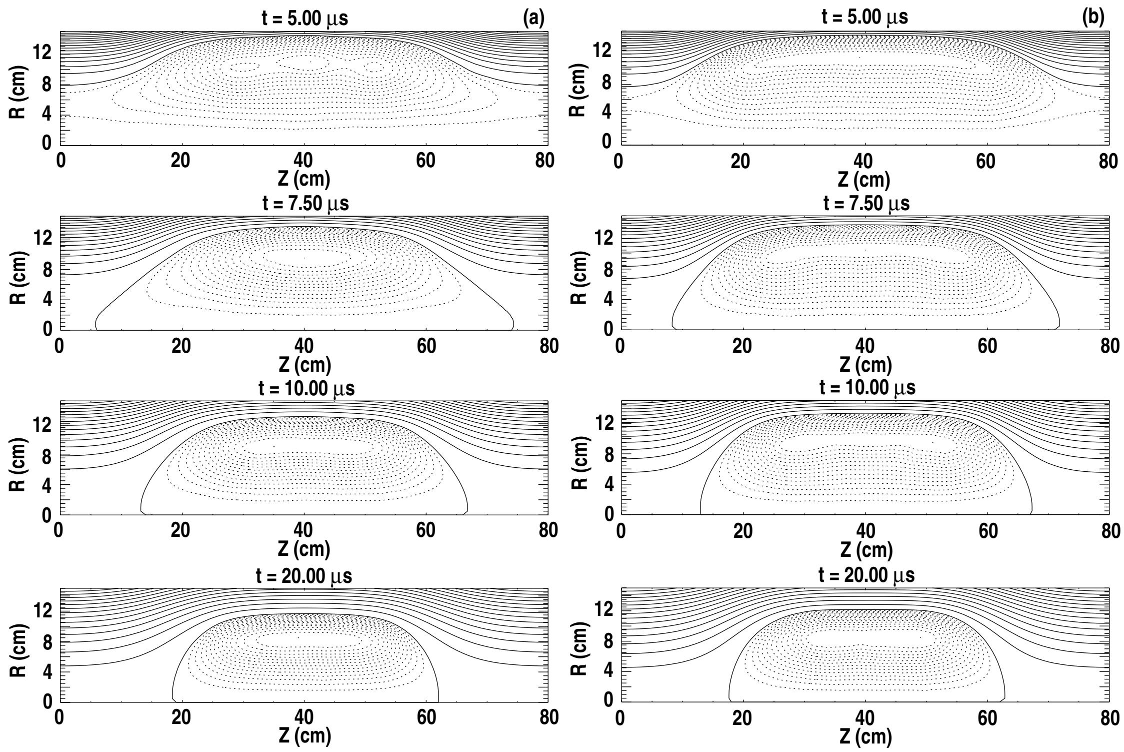

FIG. 5. Time evolution of poloidal flux function ψ (axisymmetric formation, $B_\theta \neq 0$): (a) $s^* = 3$; (b) $s^* = 12$.

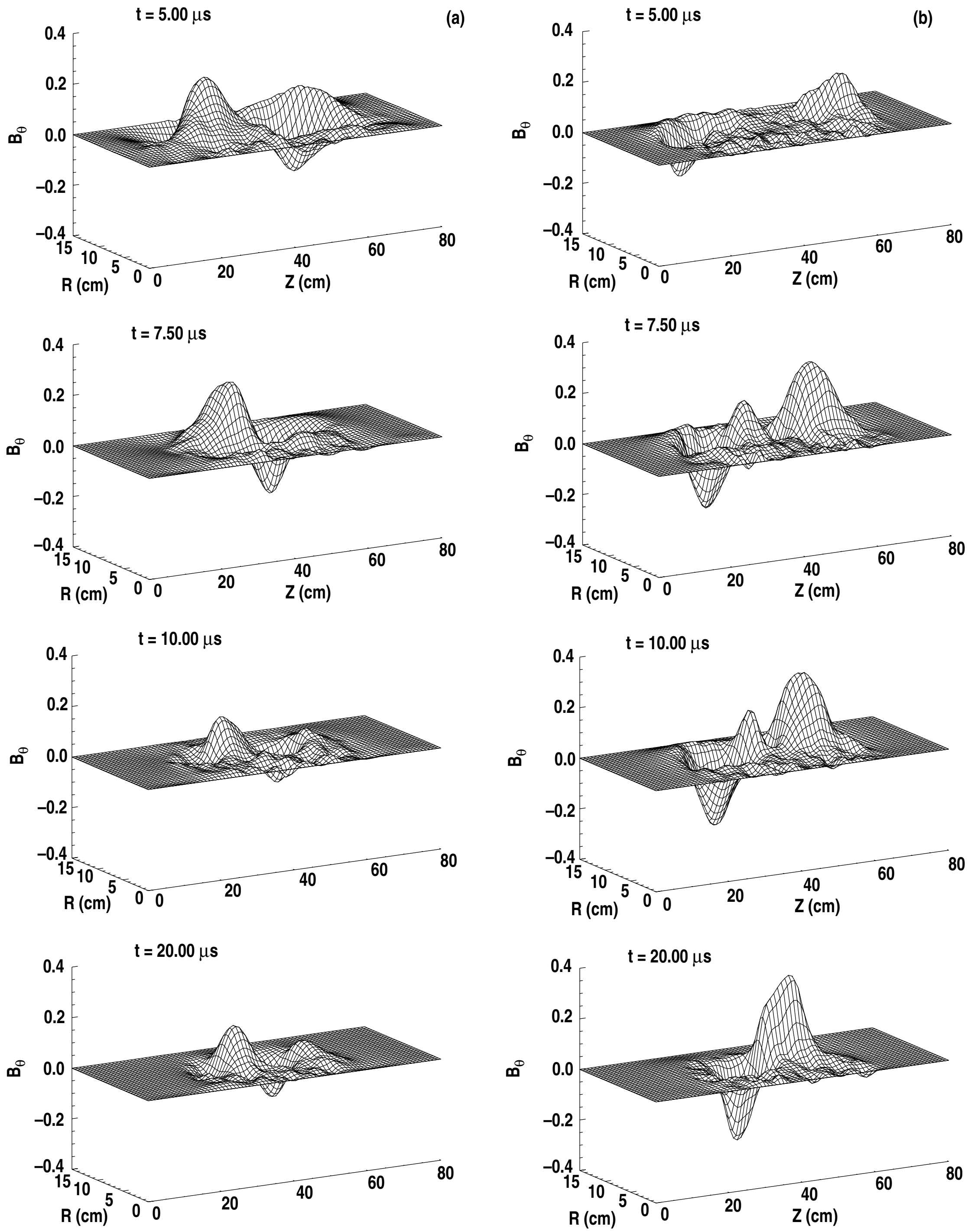

FIG. 6. Time evolution of toroidal magnetic field B_θ normalized to the separatrix midplane value of B_z (axisymmetric formation): (a) $s^* = 3$; (b) $s^* = 12$.

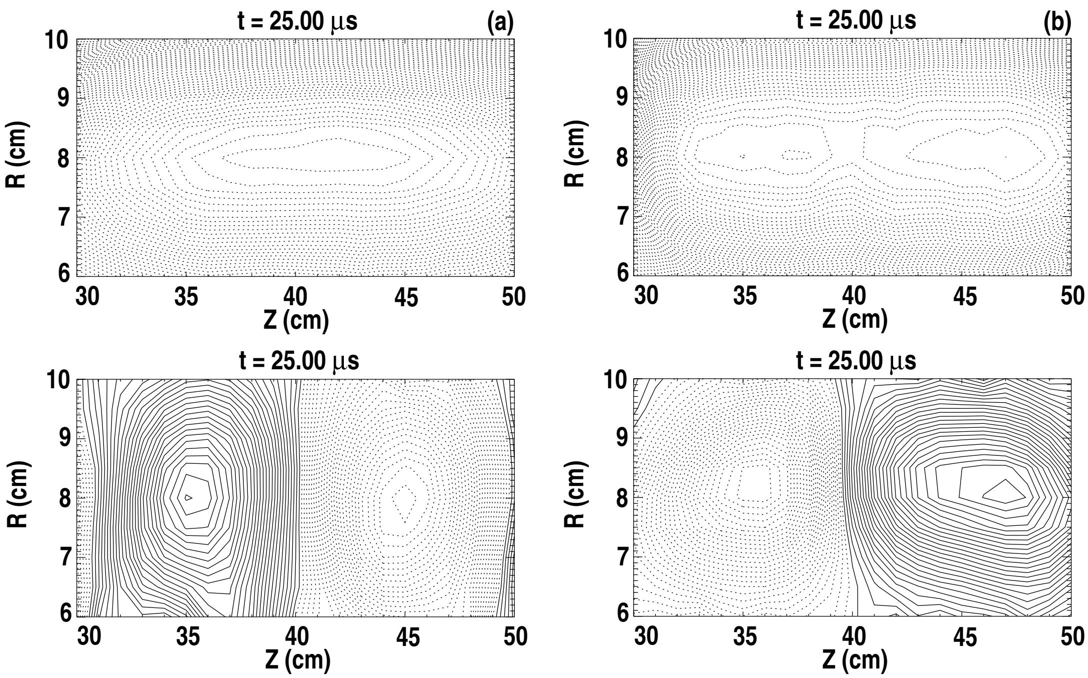

FIG. 7. Zoomed-in surface contours of ψ (above) and B_θ (below) (axisymmetric formation): (a) $s^* = 3$; (b) $s^* = 12$.

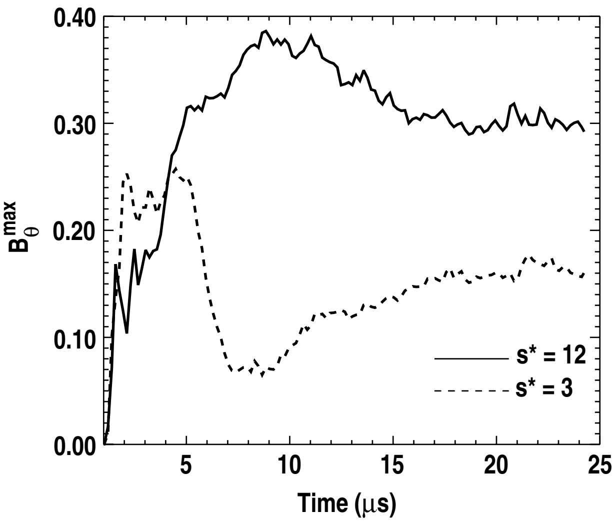

FIG. 8. Time histories of maximum B_θ normalized to the separatrix midplane value of B_z (axisymmetric formation).

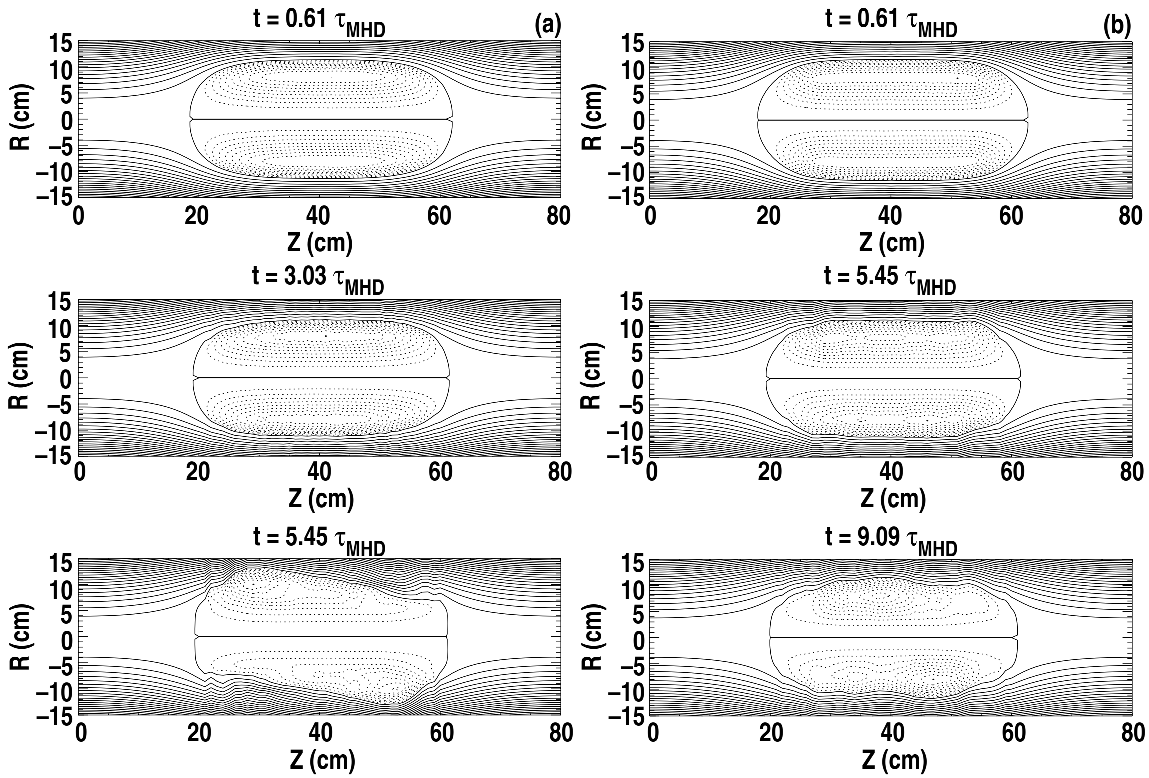

FIG. 9. Tilting ψ surfaces ($B_\theta = 0$): (a) $s^* = 3$; (b) $s^* = 12$.

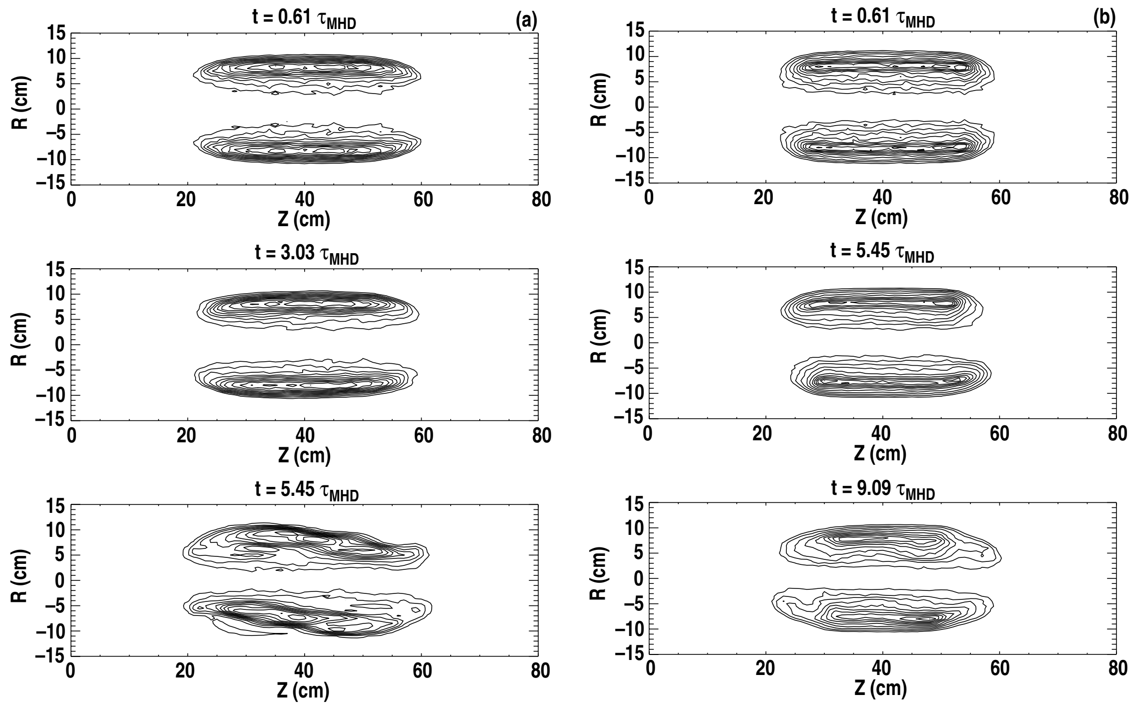

FIG. 10. Tilting plasma density surfaces ($B_\theta = 0$): (a) $s^* = 3$; (b) $s^* = 12$.

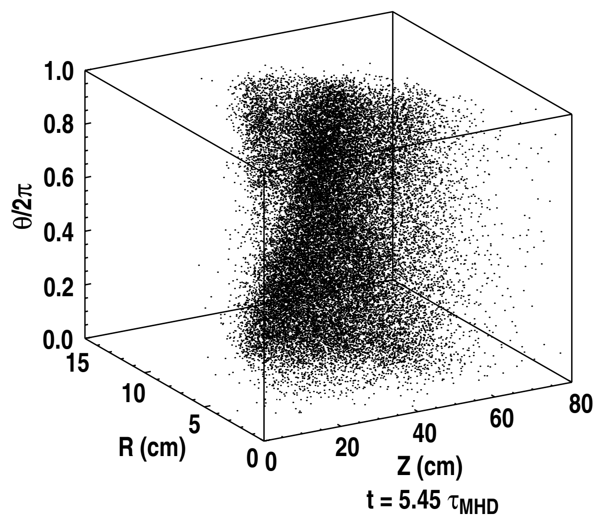

FIG. 11. Example of a 3-D tilted particle distribution ($s^* = 3$, $B_\theta = 0$).

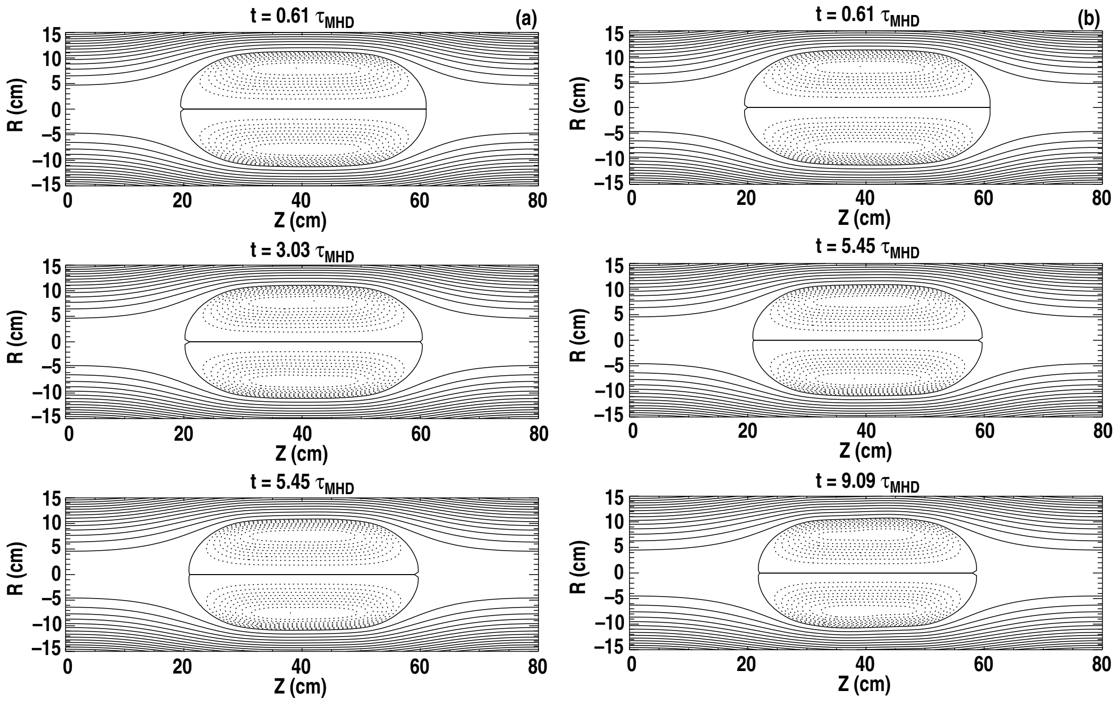

FIG. 12. Non-tilting ψ surfaces ($B_\theta \neq 0$): (a) $s^* = 3$; (b) $s^* = 12$.

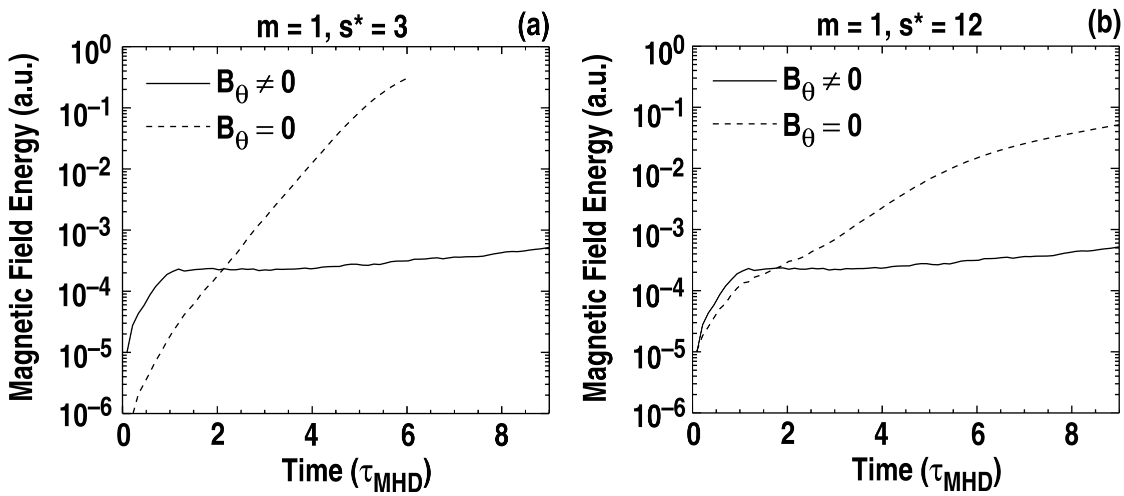

FIG. 13. Comparison of magnetic field energy in the $m=1$ tilt mode in the reduced ($B_\theta = 0$) and full ($B_\theta \neq 0$) simulations: (a) $s^* = 3$; (b) $s^* = 12$.