



**Final Report for Research on
Magnetized-Target Fusion for Space Propulsion**

John F. Santarius and Mohamed E. Sawan

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***FUSION TECHNOLOGY INSTITUTE
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1 Overview

This *Final Report* summarizes research activities at the University of Wisconsin on using a field-reversed configuration (FRC) as the target in a magnetized-target fusion (MTF) rocket. The MTF rocket is based on a concept invented by Francis Thio, in which plasmas launched by plasma guns implode an FRC or spheromak [1,2].

The four tasks defined for this research project were:

- Task 1) Participate in the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project.
- Task 2) Undertake the modeling of the implosion of a magnetized target plasma by a high-velocity plasma liner to achieve thermonuclear fusion reactions.
- Task 3) Undertake modeling the fusion burn.
- Task 4) Undertake the modeling of the expansion of the fusion plasma against a magnetic field in a magnetic nozzle configuration.

The following sections give a brief account of project activities that address the above tasks. An Appendix in the form of a Mathematica™ [3] notebook gives further details of the calculations and figures appearing in Section 2. Tasks 2 and 3 are very closely related, so they are treated in the same section (2.2).

2 Project activities

2.1 Task 1: Participate in the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project.

The main objective of the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) program is to enable future NASA missions by the development of revolutionary aerospace systems concepts and related technology. The PI for the this research participated in the April 16-17, 2002 RASC/HOPE meeting at NASA Langley Research Center. At the meeting, he provided expertise in fusion space propulsion, space resources including helium-3 fusion fuel, and advanced-fuel fusion power plant design. He has continued this task in discussions with Dr. Francis Thio during the time frame covered by this report. Advice was also provided to Dr. Slade White, NASA MSFC, regarding neutron interactions with the magnetic nozzle.

2.2 Task 2: Undertake the modeling of the implosion of a magnetized target plasma by a high-velocity plasma liner to achieve thermonuclear fusion reactions and Task 3: Undertake modeling the fusion burn.

2.2.1 Description of the BUCKY computer code

These tasks model plasma-jet magnetized-target fusion (MTF) implosion dynamics and burn dynamics using the University of Wisconsin's 1-D radiation hydrodynamics code, BUCKY [4,5]. The BUCKY code has been ported to the newly acquired fast PC cluster at the University of Wisconsin's Fusion Technology Institute, and the cases of this section were run on those computers. BUCKY is a 1-D Lagrangian radiation-hydrodynamics code which can simulate plasmas in planar, cylindrical, or spherical geometries. It solves single-fluid equations of motion with pressure contributions from electrons, ions, radiation, and fast charged particles. Plasma energy transfer can be treated using either a one-temperature ($T_e=T_i$) or two-temperature model. Both the electrons and ions are assumed to have Maxwellian distributions defined by T_i and T_e . Thermal conduction for each species is treated using Spitzer conductivities, with the electron conduction being flux-limited. The temperature equations are coupled by an electron-ion energy exchange term, and each equation has a PdV work term.

In addition to radiation, BUCKY includes a number of other physical processes as source terms in the energy equations: fast ion (beam or target debris) energy deposition; heating due to the deposition of fast charged particles and neutrons during the fusion burn phase; laser energy deposition; and x-ray heating of a cold buffer gas. Fusion burn equations for D-T, D-D, and D-³He reactions are solved, and the charged particle reaction products are transported and slowed using a time-dependent particle tracking algorithm. Neutrons are deposited in the target using an escape probability model. Fast ions from an ion beam or target microexplosion debris are tracked using a time-, energy-, and species-dependent stopping power model. Stopping powers are computed using a Lindhard model at low projectile energies and a Bethe model at high energies. Laser energy is deposited using an inverse bremsstrahlung attenuation model, with a dump of the remaining laser energy at the critical surface.

Results for this task appear in Sec. 2.2.2. The MTF rocket embodiment that served as the starting point for this work was given as paper AIAA-99-2703, by Thio, et al., originally presented at the 35th Joint Propulsion Conference [2]. That paper, in turn, is based on the models proposed in a paper by Thio, et al., published in *Current Trends in International Fusion Research—Proceedings of the Second International Symposium* [1].

Because there was not sufficient time to add a plasma-jet MTF magnetic-field model to BUCKY on the time frame of this project, the magnetic field was not included in the calculations. The neglect of the magnetic field has the drawback of not accurately simulating the thermal conductivity of the plasma, which must then be parameterized. This approximation was expected to be reasonable, because the jet kinetic energy dominates the magnetic-field pressure in the FRC target (see Appendix). For some constant value of the thermal conductivity, the total heating of the compressed target will be the same as for the case of a thermal conductivity that depends on the magnetic field. Unfortunately, the magnetic field dependence of the thermal conductivity appears to be much more sensitive to details of the compression profile and timing

than anticipated, and no suitable reference case near the parameters of Refs. 1 and 2 was found. In particular, for a constant thermal conductivity, unrealistically little heat is lost from the plasma during the early stages when the magnetic field is low, so the relatively high pressure reduces the compression. Perhaps even more important is that the lack of a magnetic field means that alpha particles generated by D-T reactions will deposit their energy over a much larger distance than if they were constrained by a finite gyroradius.

Progress to date has been good, with a framework set up to examine plasma physics aspects of MTF cases, many parametric cases run and detailed information from these cases available in easily analyzed forms, as shown in the figures of Sec. 2.2.2. Nevertheless, further work, primarily modifying the BUCKY code to include the magnetic field and the dependence of the plasma thermal conductivity on it, will be required to reproduce the base case outputs in detail. In particular, the right balance between compression and heating has not yet been found that would give the same fusion output power as in the base cases. Much more detail for the present calculations and the output for some of the parametric variations appears in the accompanying Mathematicatm notebook of the Appendix. This notebook was created during the course of this project in order to analyze plasma-jet MTF plasma physics, generate BUCKY input parameters, and post-process the BUCKY results.

2.2.2 Results

2.2.2.1 Reference case

The input parameters that formed the basis of parametric runs searching for a case with output near that of paper Refs. [1] and [2] appear in the Appendix subsection titled *Parameters based on Thio, et al, Current Trends in International Fusion Research 2nd Symposium and Thio, et al, AIAA-99-2703*. Note that $v_{jet}=125$ km/s for this case, which served as the basis for a search through parameter space for a high-Q (fusion power/input power) case. The difficulties in the search stem from the fact that the timing of the shock waves depends strongly on the plasma parameters and dimensions, while the fusion power produced depends in turn on that timing. As discussed in the previous section, the BUCKY code does not yet treat the magnetic field, and the relation between thermal conductivity and magnetic field is particularly important, as is the effect of the magnetic field on localizing the deposition of alpha particle energy. The cases run so far have parameterized the thermal conductivity in order to model this effect approximately. Also, differences exist between the ideal-gas analytic model of Refs. [1] and [2] compared to the BUCKY calculations, which include the full deuterium and tritium equations of state. The analytic model indicates that an attractive solution exists, but the difficulty in finding this solution with BUCKY shows that improvements in the BUCKY physics must still be made.

Two key characteristics of the physics of magnetized target fusion are the amount of compression and the dwell time during which the target plasma remains compressed. The amount of compression relates to the target plasma density and temperature, of which the fusion power is a strong, nonlinear function. Typical Lagrangian zone boundary radii versus time ($0 < t < 2 \mu s$) for the reference case, based on Refs. [1] and [2], appear in Figure 1; the inset shows the key parameters. After the initial shock caused by the jets impacting the target, the jet plasma rebounds and interacts with the somewhat slower incoming buffer plasma. The buffer plasma's

role is to mitigate the neutron energy. A rebounding shock in the target sends an outgoing shock through the buffer, starting at $\sim 0.5 \mu\text{s}$. The jet plasma rebounds from the buffer and meets the expanding target plasma, causing the target to compress again. For maximum effectiveness, all of the shock waves, which appear as localized compression of lines in the figure, would converge, so this case is clearly not optimized. The evolution of the same shock wave, for times $< 0.4 \mu\text{s}$ and radii $< 0.05 \text{ m}$, appears in Figure 2.

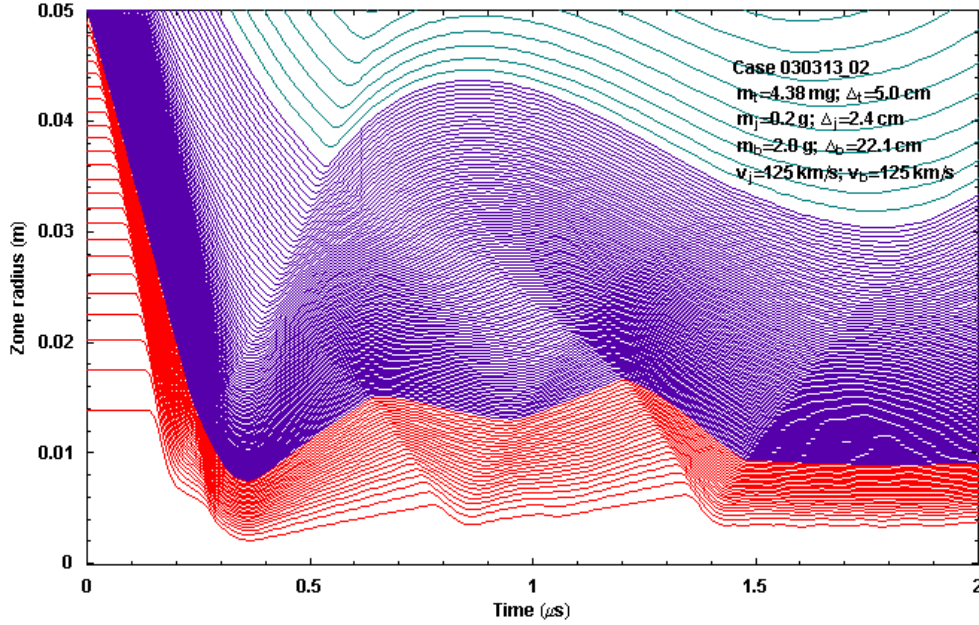


Figure 1. Zone boundary radii vs. time from 0-2 μs for a $v_{\text{jet}}=125 \text{ km/s}$ case. Target zones are red, jet zones are violet, and buffer zones are green.

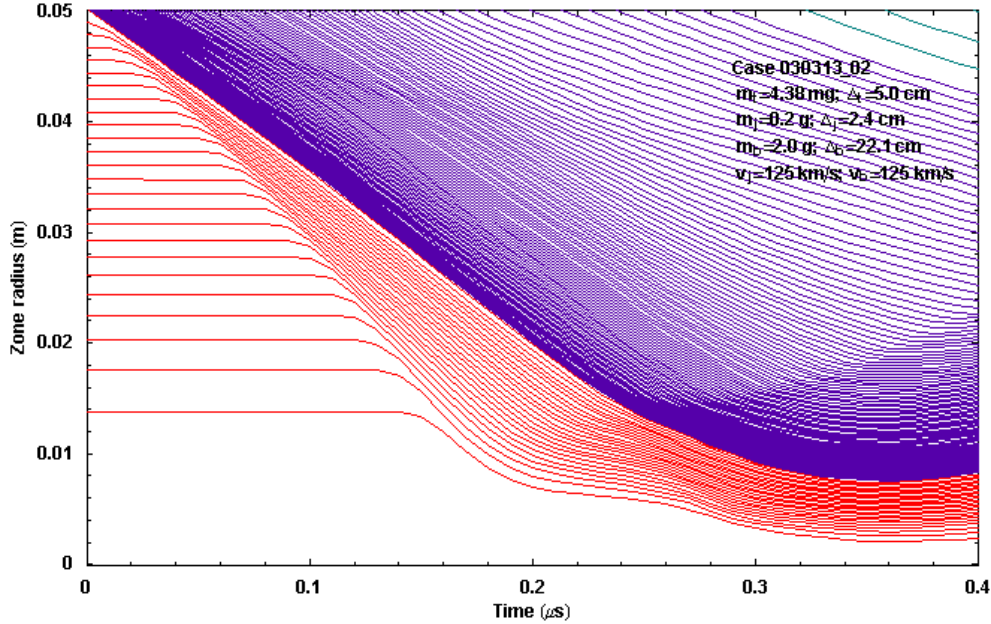


Figure 2. Zone boundary radii vs. time from 0-0.4 μs for a $v_{\text{jet}}=125 \text{ km/s}$ BUCKY case. Target zones are red and jet zones are violet.

In the parametric exploration runs, essentially all of the fusion power is generated during the initial target compression, and only a very small amount is generated during the secondary compressions. This cycle repeats until eventually all three plasma regions expand outward. The density and temperature for each region versus time appear in Figure 3 and Figure 4. BUCKY also can generate a wide variety of parameters versus other parameters at selected times, and some of these cases appear in the Appendix. Cases for various other plasma target and jet conditions also appear in the Appendix. Many cases have been run in an attempt to optimize the parameters, but the reference case [1] has not yet been reproduced. Related considerations are discussed below.

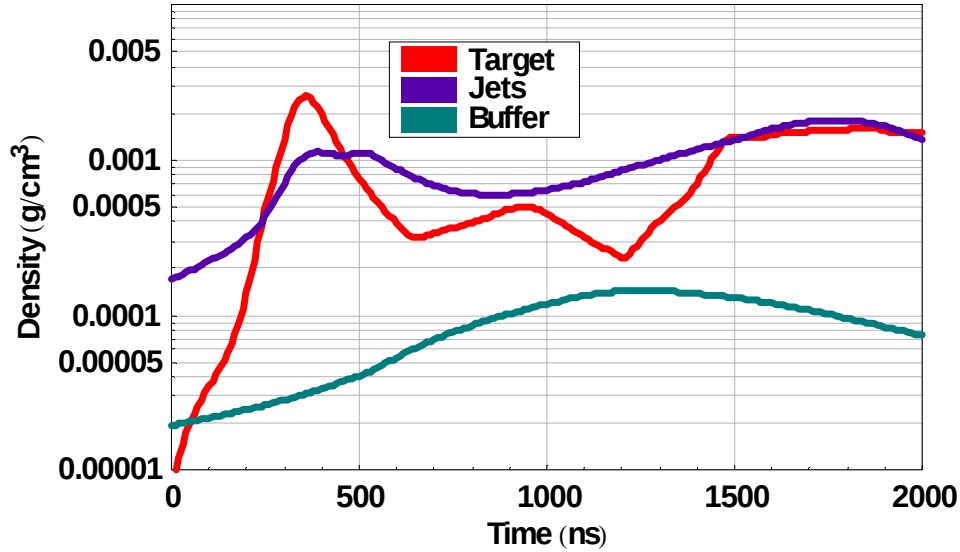


Figure 3. Density vs. time for a $v_{\text{jet}}=125$ km/s BUCKY case.

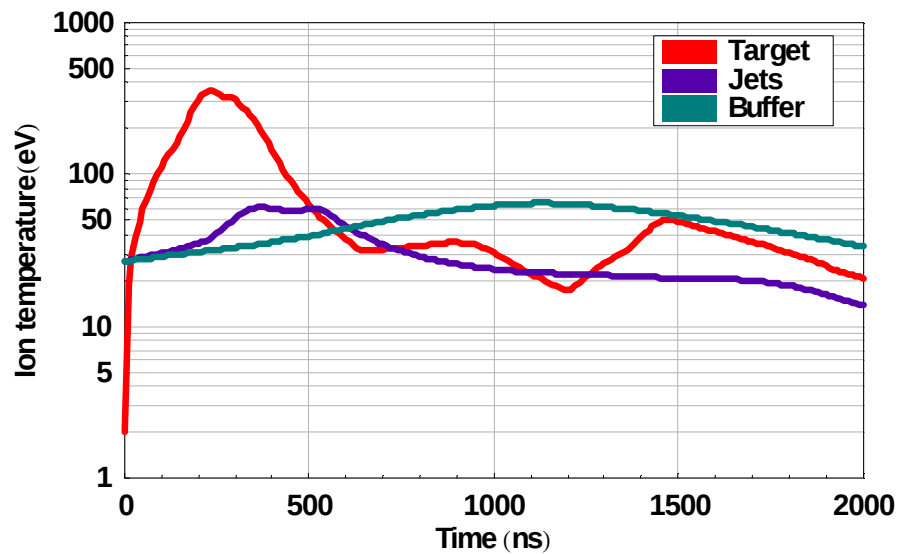


Figure 4. Ion temperature vs. time for a $v_{\text{jet}}=125$ km/s BUCKY case.

2.2.2.2 Case with jet velocity increased to 398 km/s

Because it proved difficult to reproduce Thio, et al.'s reference case without a magnetic field in the BUCKY code, the decision was made to explore a high-velocity plasma-jet case, nominally 400 km/s. A jet velocity of 398 km/s was used in order to satisfy a “perfect acoustic matching” condition [1]. When this condition applies, the velocity of the contact surface between the jets and the target equals the ion-acoustic velocity in the jets, resulting in no outward radial shock propagation in the jets.

Typical Lagrangian zone boundary radii versus time for this case appear in Figure 5, which shows the first 0.2 μs for radii < 0.05 m. The momentum of the jets now suffices to overwhelm most of the shocks, giving good inertial confinement for ~ 100 ns. The compression can be seen to be a maximum at $\approx 0.15 \mu\text{s}$ and to expand slightly beyond that time. The evolution of the same shocks appears in Figure 6 with the y axis expanded so that $r < 0.02$ m; it shows more numerous shocks with smaller amplitudes than in the reference case, but shocks have not been completely eliminated.

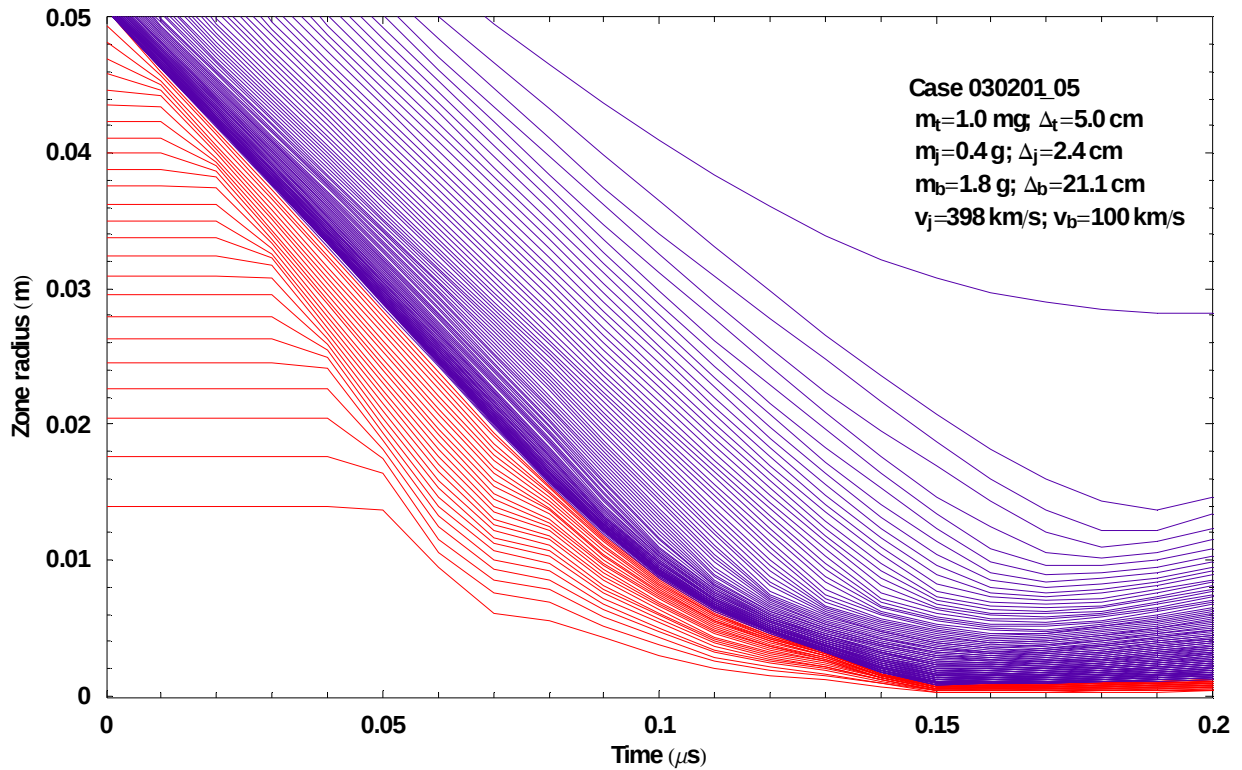


Figure 5. Zone boundary radii ($r < 0.05$ m) vs. time (0-0.2 μs) for a $v_{\text{jet}}=398$ km/s case. Target zones are red and jet zones are violet.

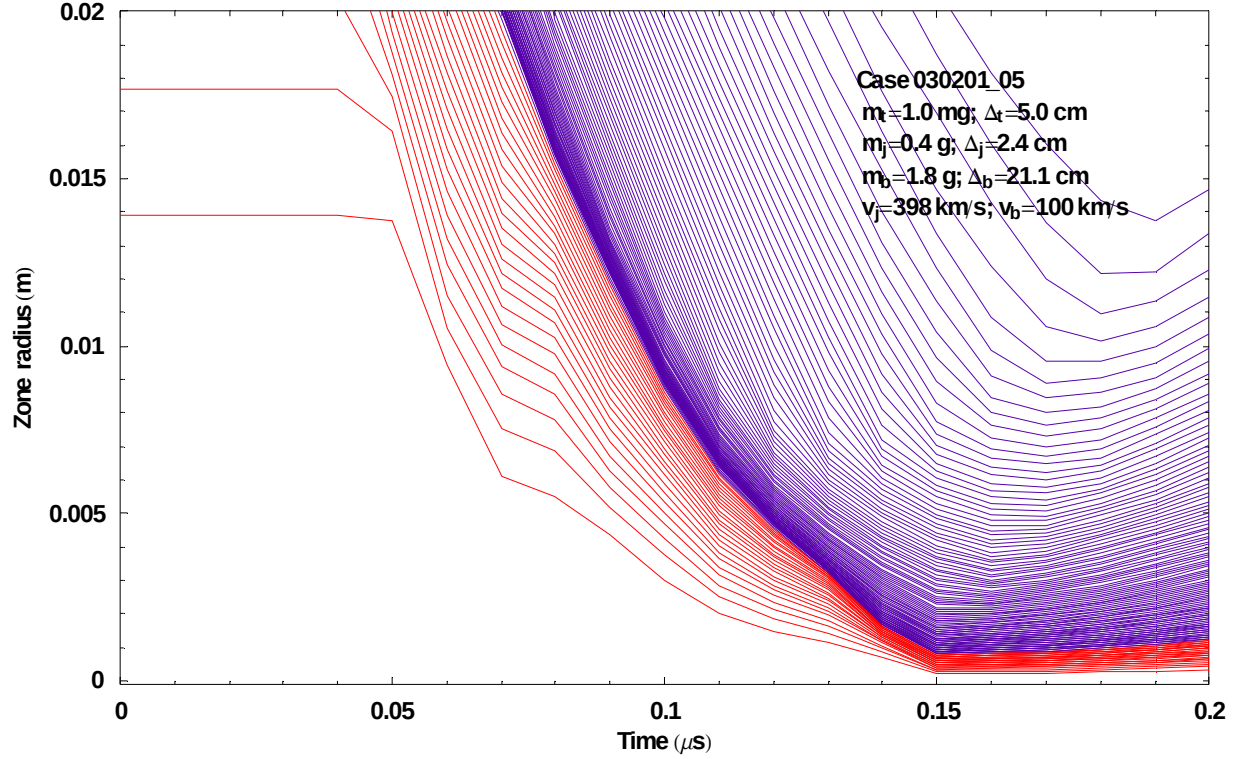


Figure 6. Zone boundary radii ($r < 0.02$ m) vs. time (0-0.2 μ s) for a $v_{\text{jet}} = 398$ km/s BUCKY case. Target zones are red and jet zones are violet.

The neutron production averaged over the various regions versus time appears in Figure 7, which shows that most of the neutron production takes place over the relatively short time span of ~ 30 ns. The total fusion power produced for this case was only about 700 kJ, much less than expected from Refs. 1 and 2. The explanation for this difference appears to be the timing of the density and temperature peaks for each region as illustrated in Figure 8 and Figure 9 (averaged over each region). The density peaks about 30 ns later than the ion temperature. This seems likely to be due to the differences between the analytic and computer-code models, probably the lack of dependence of the thermal conductivity and alpha particle energy deposition on magnetic field in the BUCKY computer code. This leads to less compression of the plasma at early times when the magnetic field is low in the computer calculations, because the assumption of high, constant thermal conductivity leads to a higher temperature and, therefore, higher pressure than in the analytic model. The mass density, ion temperature, and specific fusion power for the individual Lagrangian zones appear in Figure 10 for times from 60-160 ns at 20 ns intervals. The enhancement of the alpha particle energy deposition by magnetic fields of the order of 500 T to 1000 T over the size of the compressed target of a few millimeters can be significant due to the small gyro-radius of the alpha particles. This enhancement is not incorporated in the BUCKY calculation at this point. The enhanced alpha energy deposition could potentially maintain the target temperature at the elevated level or even ramp up the temperature.

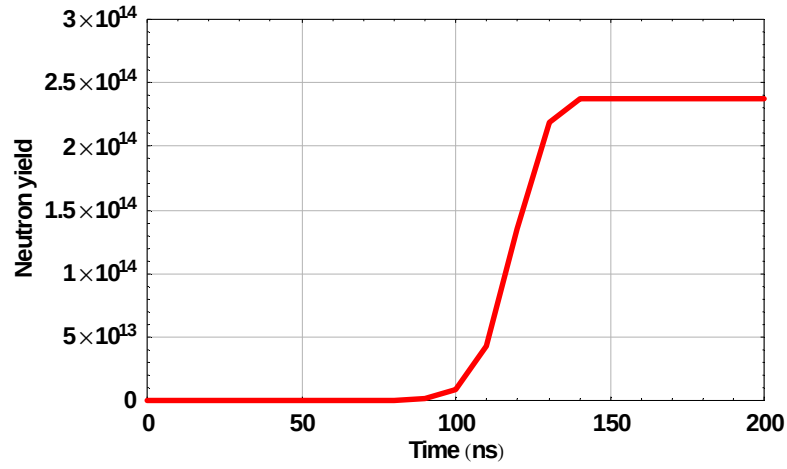


Figure 7. Neutron production vs. time for a $v_{\text{jet}}=398$ km/s BUCKY case.

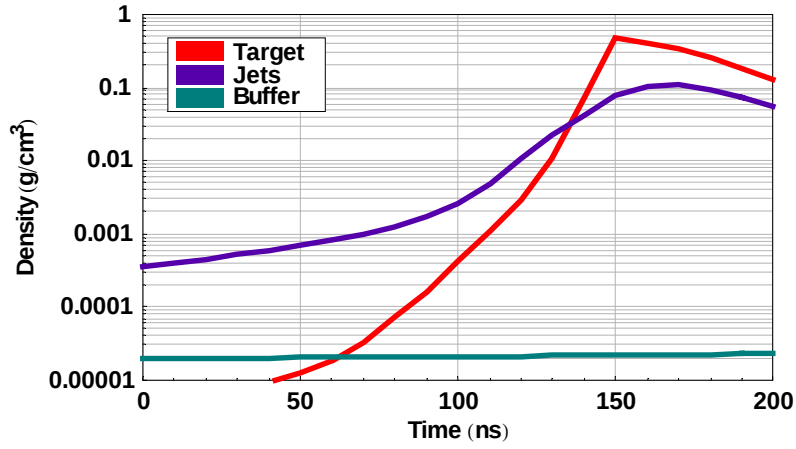


Figure 8. Density vs. time for a $v_{\text{jet}}=398$ km/s BUCKY case.

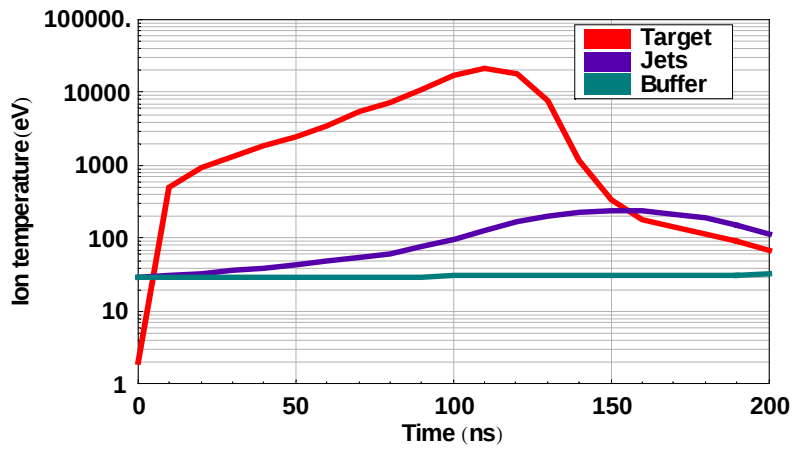


Figure 9. Ion temperature vs. time for a $v_{\text{jet}}=398$ km/s BUCKY case.

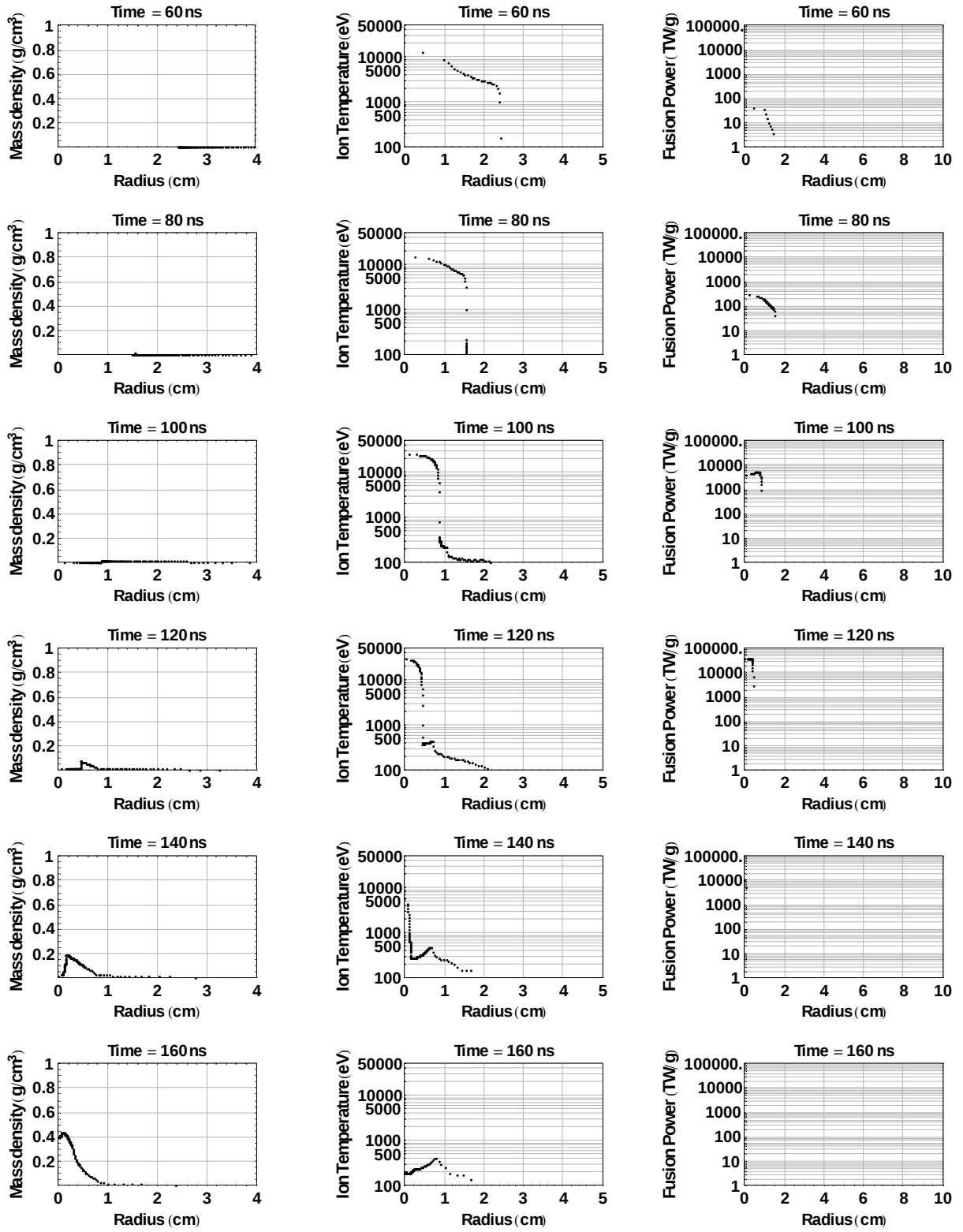


Figure 10. Mass density, ion temperature, and specific fusion power vs. radius near the origin from 60-160 ns for a $v_{\text{jet}}=398$ km/s BUCKY case. Each dot represents a Lagrangian zone.

2.3 Task 4: Undertake the modeling of the expansion of the fusion plasma against a magnetic field in a magnetic nozzle configuration.

The key foci of this activity were: (1) searching the literature for support of the 80% predicted nozzle efficiency; (2) calculating the energy produced in the nozzle magnets and structure by neutrons and gamma rays (Appendix subsection *Neutronics/MES ONEDANT cases*); and (3) setting up functions to calculate the neutron moderation by the buffer for use when a suitable BUCKY case has been found (Appendix subsection *Neutronics/Calculation of neutron slowing down in deuterium*).

Investigating the engineering and scientific literature related to magnetic nozzles for pulsed plasma propulsion led to the conclusion that the predicted nozzle efficiencies (up to ~80%) appear to be achievable [6,7,8,9,10,11,12,13,14,15,16]. Some detailed inertial-confinement fusion calculations predict 65% efficiency [8,13]. The projected specific powers of plasma MTF propulsion systems are so high that even a reduction of the predicted 80% nozzle efficiency by a factor of two would leave sufficient margin to perform well for most missions of interest.

The key nozzle questions from the fusion technology standpoint are:

1. What fraction of the fusion neutron energy would be deposited in the nozzle?
2. What would be the consequent heating of the structure?

Dr. Mohamed E. Sawan of the UW Fusion Technology Institute agreed to perform some quick calculations, using the ONEDANT module of the DANTSYS 3.0 discrete ordinates particle transport code system [17] to perform calculations to indicate the approximate amount of the heating starting with various neutron energies. The most recent version of the International Fusion Energy Nuclear Data Library, FENDL-2 [18], was utilized in a 175 neutron energy group structure. For this purpose, 1-D geometry and a 5-cm thick slab of 316 stainless steel served to approximate the magnetic coil and structure of the nozzle. The cross sections for reactions with neutrons of most candidate materials are sufficiently close to those of steel that the results should be broadly applicable. Materials also can generally be tailored to reduce heating by specific isotopes by avoiding those isotopes or reducing their amount.

The 5 cm magnet coil thickness stems from assuming an average coil current density for the superconductor, stabilizer, and structure of 50-100 MA/m². Performance of 50 MA/m² has been achieved for a twisted, 6.9 T magnetic coil of inner dimension 3.9 m [19], so this assumption seems reasonable for a zero order estimate.

The crucial neutronics issue facing magnetic nozzle designers is that the absorption of even very low energy neutrons can lead to (n,γ) reactions that produce several MeV of energy per reaction. Fortunately, the mean free path for neutrons in materials can be much larger than the structure thickness, depending on the neutron's energy, so most high-energy neutrons will pass through the few cm's of structure. The deposited neutron and gamma-ray energy as a function of the initial neutron energy are shown in Figure 11, and the resulting neutron energy multiplication appears in Figure 12. The energy absorbed by the nozzle magnet and structure as

a fraction of the initial fusion energy appears in Figure 13. As Figure 13 illustrates, there exists a broad minimum in the absorbed energy by the nozzle of less than 1% of the fusion energy over a fairly wide range of initial neutron energies, 200-855 keV. A buffer plasma of properly chosen parameters should be able to limit almost all of the neutrons exiting it to this energy range.

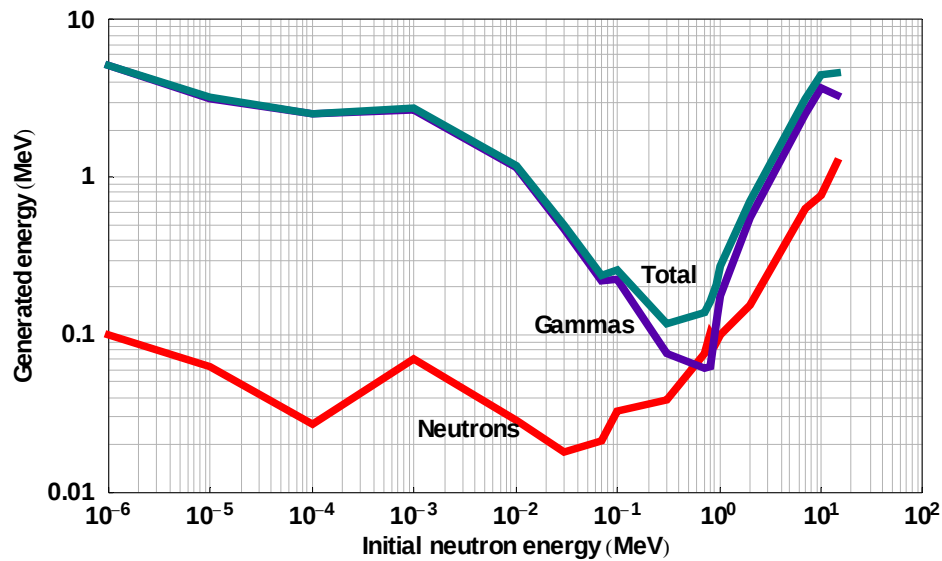


Figure 11. Energy deposited in 5 cm of 316 stainless steel versus initial neutron energy. Energy from neutrons in red, from gamma rays in violet, and total in green.

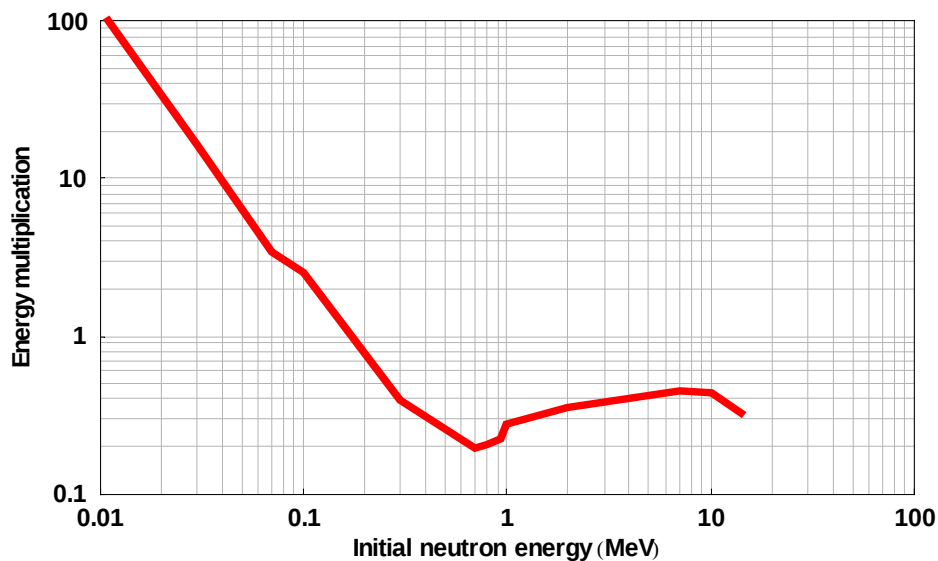


Figure 12. Energy multiplication in 5 cm of 316 stainless steel as a function of initial neutron energy.

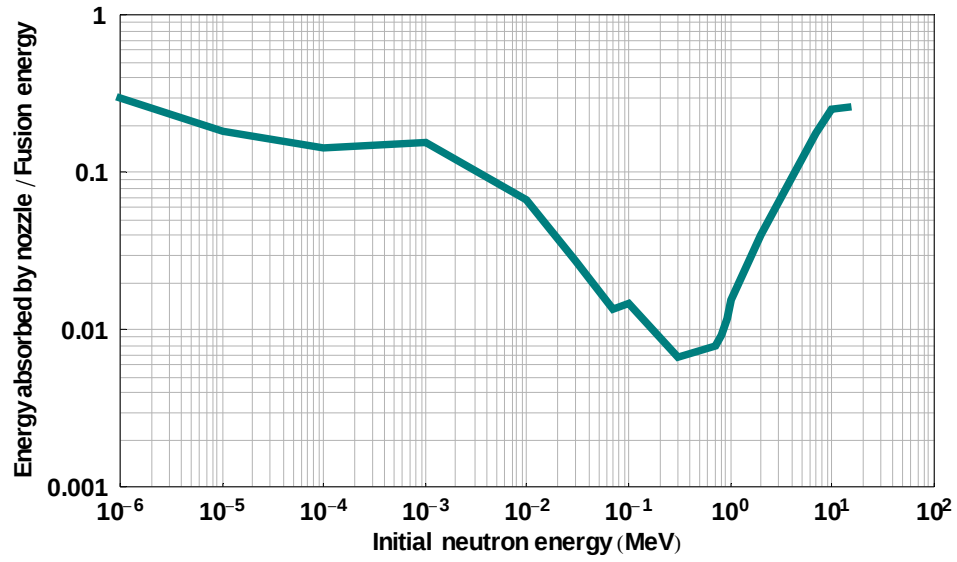


Figure 13. Energy absorbed by 5 cm of 316 stainless steel as a fraction of the total fusion energy for a given initial neutron energy.

3 Summary

This research supported NASA's Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project by providing expertise on fusion propulsion and evaluating technical aspects of plasma-jet magnetized-target fusion (MTF). The 1-D radiation hydrodynamics of the plasma-jet induced implosion/explosion of the target plasma was modeled using the University of Wisconsin's BUCKY computer code [4]. The calculations qualitatively reproduce the anticipated behavior of the plasma regions, but the desired reference case [1,2] remains elusive. Further exploration of operating parameter space may reproduce this case, but it appears more likely that the discrepancy results from differences in the assumptions used. In particular, the BUCKY code does not include magnetic fields, and thereby neglects the localization of alpha-particle heating and the relationship between thermal conductivity and magnetic field, which will likely be very important factors. Also, the analytic model of Refs. 1 and 2 uses ideal-gas equations of state, whereas the BUCKY calculations use table lookup for the full deuterium and tritium equations of state and opacities. This is not, however, expected to be an important difference. The BUCKY computer code also includes independent ion and electron temperatures. This research also numerically assessed the heating of the structure and magnets of the magnetic nozzle by neutrons from the fusion burn. The resulting nozzle heating could be kept below 1% of the total fusion power by using a plasma liner to buffer the neutron energy into the 200-855 keV range. This assumes that the total thickness of the magnet's superconductor, stabilizer, structure, and shielding would be less than ~ 0.05 m, a value consistent with presently achieved superconducting magnet winding pack current densities.

4 Recommendations for Future Work

Several extensions to this research would improve the accuracy of plasma-jet MTF modeling and optimize design points for this concept:

1. Implement a magnetic-field dependent thermal conductivity and alpha-particle energy deposition in BUCKY and use that code to
 - a. reproduce the published plasma-jet MTF analytic cases and
 - b. optimize the burn dynamics of plasma-jet MTF.
2. Implement and test FRC equilibrium and stochastic magnetic field options in a 2-D particle-in-cell or discrete simulation Monte Carlo code. Use the chosen code to analyze alpha-particle transport in plasma-jet MTF.
3. Find optimized plasma-jet MTF cases for a range of experiments along the development path to a space-qualified fusion propulsion system.
4. Calculate the energy produced in the magnetic nozzle by neutrons and gamma rays for the full spectrum of neutron energies produced by the MTF fusion burn.

5 Acknowledgments

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Appendix: Final Report Magnetized-Target Fusion for Space Propulsion

Mathematica Notebook

FinalReport_PlasmaMTF_03_Appendix.nb

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Introduction

This notebook contains calculations related to plasma-jet magnetized-target fusion (MTF), in which a magnetized plasmoid is compressed by plasma jets.

Units are SI, except that energies and temperatures are usually in eV or keV.

JFS

6/17/03

17:03:56

Notes and notation

■ Notes

- ★ *Note: spherical geometry used for BUCKY 1-D radiation hydrodynamics calculations.*
- ★ *Note: the densities in some plasma regimes are so high and the temperatures are so low that the Coulomb logarithms are very small (<5). Care must be exercised in interpreting results related to collisions.*
- ★ *Note: presently assuming fully ionized deuterium-tritium for target and jets.*

■ Neglecting magnetic field

The fastest initial interaction as the jets meet the target will be between the jet and target electrons. This will tend to equilibrate the electron temperatures, perhaps through a two-stream instability. Because the jet electrons have a drifted Maxwellian distribution with its peak at $\epsilon_{ej} < 1 \text{ eV} \ll \frac{3}{2} T_e$, this should be a small effect. Any resulting electrostatic fluctuations will be transmitted to the ions on the ion-electron collisional time scale.

The target and undrifted jets, with $T \sim 2 \text{ eV}$ and $n \sim 10^{22} - 10^{24} \text{ m}^{-3}$, have internal mean free paths of 3-15 μm . This should mean that the internal dynamics can be separated from the interaction dynamics to a large extent, so the first calculations will neglect the internal scales.

The main momentum transfer should occur through slowing down of jet ions on target electrons and the resulting electrostatic pull on target ions. Plasma oscillations and possible instabilities may then be a factor. Shock waves will be induced. The key length scales should be the ion-electron slowing down mean free path and the ion-ion angle scattering mean free path.

JFS

6/17/03

17:03:56

■ Running BUCKY MTF cases on ZEEP PC computer cluster

Run X Windows (xwin32).

Log onto ZEEP using SecureCRT.

Start Emacs.

Modify input case: e.g., mtf_020807_01.inp

Start a shell within Emacs; run the command, e.g.,

```
qsub -v job=020807_01 bucky.sh
```

Results will be in the zeep directory bucky/cases/020708_01/.

File transfer can be done using sftp.

■ Post-processing BUCKY/BPLOT files

The following BPLOT files are presently post-processed in this Mma notebook:

BPLOT.special (processed into plot.pfus.r.dat)

PLOT.mass.vel

PLOT.NeutronsYield.vs.t

PLOT.regn.dens

PLOT.regn.KEs

PLOT.regn.temp

PLOT.rho

PLOT.tempn

PLOT.vel

PLOT.zones.1

PLOT.zones.2

The first step is to change fortran exponential notation to Mma 10^x notation. This can be done using sed:

```
...
sed 's/E+/*^/g' PLOT.vel |sed 's/E-/*^-/g' > plot.vel.dat
cat PLOT.zones.1 PLOT.zones.2 > PLOT.zones
sed 's/E+/*^/g' PLOT.zones |sed 's/E-/*^-/g' > plot.zones.dat
```

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rm PLOT.zones

For the files listed in the previous paragraph, these commands are contained in the file ExpToMma.sed. The command "sh ExpToMma.sed" in a Cygwin window processes the files. Commands to process only the files from bptec4 generated at discrete times are in the file ExpToMmaTimes.sed.

■ Notation

■ Subscripts

b $\equiv$ buffer (neutron-moderating layer of liner)

c $\equiv$ compressed target

e $\equiv$ electron

i $\equiv$ ion

j $\equiv$ jet (inner layer of liner)

t $\equiv$ target

w $\equiv$ wall

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■ General parameter definitions

<i>Symbol</i>	<i>I/O Unit</i>	<i>Working Unit</i>	<i>Definition</i>
A_j	m^2	m^2	Total jet inner area at target impact
B_t	T	T	Target magnetic field
Δ_j	m	m	Jet pulse thickness
ε_{eb}	eV	eV	Buffer electron average energy
ε_{ib}	eV	eV	Buffer ion average energy
ε_{ec}	eV	eV	Compressed target electron average energy
ε_{ic}	eV	eV	Compressed target ion average energy
ε_{ej}	eV	eV	Jet electron average energy
ε_{ij}	eV	eV	Jet ion average energy
ε_{et}	eV	eV	Target electron average energy
ε_{it}	eV	eV	Target ion average energy
$\mathcal{E}_{ej}$	eV	eV	Jet electron directed energy
$\mathcal{E}_{ij}$	eV	eV	Jet ion directed energy
f_{rep}	Hz	Hz	Repetition rate
Γ_j	$m^{-2}s$	$m^{-2}s$	Jet ion flux
γ_t			Target gamma
M_j	mg	kg	Per jet mass
m_b	g	kg	Buffer mass
m_j	mg	kg	Total jet mass
m_t	mg	kg	Target mass
N_j			Number of jets
N_b			Number of zones in buffer
N_j			Number of zones in jet
N_t			Number of zones in target
μ_b	amu	amu	Buffer atomic mass
μ_j	amu	amu	Jet atomic mass

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μ_t	amu	amu	Target atomic mass
n_b	m^{-3}	m^{-3}	Buffer number density
n_j	m^{-3}	m^{-3}	Jet number density
n_t	m^{-3}	m^{-3}	Target number density
Q_b	J	J	Buffer initial energy
Q_j	J	J	Jet initial energy
ρ_b	g/cm^3	kg/m^3	Buffer mass density
ρ_j	g/cm^3	kg/m^3	Jet mass density
ρ_t	g/cm^3	kg/m^3	Target mass density
<i>Symbol</i>	<i>I/O Unit</i>	<i>Working Unit</i>	<i>Definition</i>
r_b	m	m	Radius of outside edge of buffer
r_j	m	m	Radius of outside edge of jets
r_t	m	m	Radius of outside edge of target
r_w	m	m	Radius of inner edge of wall (distance to closest B-field coil)
T_{eb}	eV	eV	Buffer electron average temperature
T_{ib}	eV	eV	Buffer ion average temperature
T_{ec}	eV	eV	Compressed target electron average temperature
T_{ic}	eV	eV	Compressed target ion average temperature
T_{ej}	eV	eV	Jet electron average temperature
T_{ij}	eV	eV	Jet ion average temperature
T_{et}	eV	eV	Target electron average temperature
T_{it}	eV	eV	Target ion average temperature
τ_b	s	s	Buffer layer travel time to origin
τ_j	s	s	Jet layer travel time to origin
τ_{sb}	s	s	Buffer ion acoustic wave travel time across region
τ_{sj}	s	s	Jet ion acoustic wave travel time across region
τ_{st}	s	s	Target ion acoustic wave travel time across region

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v_j	km/s	m/s	Jet velocity
v_{st}	m/s/m/s		Sound (ion-acoustic) velocity in target
V_j	m^{-3}	m^{-3}	Total jet volume
ζ_b			Buffer mass zone increment
ζ_j			Jet mass zone increment
ζ_t			Target mass zone increment
z_b			Buffer ion charge state
z_c			Compressed target ion charge state
z_j			Jet ion charge state
z_t			Target ion charge state

■ BUCKY-specific definitions

	<i>Input</i>	<i>Bucky</i>	<i>Mma calc</i>	
<i>Parameter</i>	<i>Unit</i>	<i>Unit</i>	<i>Unit</i>	<i>Definition</i>
Δ_{region}	m	cm	m	Widths of regions
δ_{zone}	m	cm	m	Widths of individual zones
N_r				Total number of regions
N_z				Total number of zones
r_{region}	m	cm	m	Outer radius of regions

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■ Package *plasma.m* functions

Note: run the Preliminaries section before running these commands.

? plasma`*

AlfvenVelocity	Ecrit	ionThermalVelocity	nuieEnergy
alphasq	electronGyroFrequency	lsParams	nuiePar
beta	electronGyroRadius	lsParamsB0	nuiePerp
ChildLangmuirIspherical	electronPlasmaFrequency	lsParamUnits	nuieSlow
CoulogEE	electronThermalVelocity	lsParamUnitsB0	nuiiPerp
CoulogEI	ionAcousticVelocity	lsParamUnitsB0Hz	plasma
CoulogIE	ionGyroFrequency	lsParamUnitsHz	plasmaParams
CoulogII	ionGyroRadius	nueePerp	plasmaParamsHz
DebyeLength	ionPlasmaFrequency	nueeSlow	

? plasmaParams

```
plasmaParams[amu,B,Ee,Ei,gamma,ne,ni,Te, Ti,zi]
generates a list of plasma parameters.
plasmaParams[amu,B,Ee,Ei,gamma,ni,Te, Ti,zi]
calculates ne=ni*zi and generates a list of plasma parameters.
```

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Preliminaries

■ Mathematica setup

C:\Program Files\Common Files\Mathematica\4.0\FrontEnd\init.m (*Mathematica* initializations) and F:\Mathematica\Configuration\Kernel\init.m (my initializations) are run when *Mma* starts. My *init.m* adds package paths, turns off some error messages, sets the default font and table spacing, and invokes the *CleanSlate* package.

```
Off[System`Verbose::shdw];
```

■ Graphics

```
SetOptions[{Plot, ListPlot}, Frame → True,
  GridLines → {{Automatic, GrayLevel[0.7]}, {Automatic, GrayLevel[0.7]}}];
optionsListPlot = Options[ListPlot];
```

Convenience definition:

```
plotAll = PlotRange → All;
frameticksLog = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -30, 30}];
frameticks10 = Table[{10^i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -30, 30}];
```

FrameTicks usage: FrameTicks → {Automatic, frameticks10, None, None}

■ Input graphics package needed for many plots

```
Needs["Graphics`Graphics`"];
Needs["ExtendGraphics`Ticks`"];
Needs["Graphics`Legend`"];
```

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```
SetOptions[{LogPlot, LogLogPlot, LogListPlot, LogLogListPlot},
  Frame → True, FrameTicks → {Automatic, Automatic, None, None},
  GridLines → {{Automatic, GrayLevel[0.7]},
    {Automatic, GrayLevel[0.7]}}];
SetOptions[LogLogListPlot, PlotJoined → True];
optionsLogListPlot = Options[LogListPlot];
```

■ PlotStyles

★ *Note: the order of the colors in colorStyle is changed in this notebook compared to most others.*

Define *lineStyle* depending on whether color or dashing is desired. Use PlotStyle->lineStyle[[1]] or dheStyle, etc. kLineStyle="color" or "dashing".

```
kLineStyle = "color";
thickness = Thickness[0.01];
thicknessOrig = thickness;
ptSize = PointSize[0.005];
ptSizeOrig = ptSize;
```

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```

blue = RGBColor[0, 0, 1];
red = RGBColor[1, 0, 0];
green = RGBColor[0, 0.7, 0];
rose = RGBColor[0.667, 0, 0.333];
orange = RGBColor[0.75, 0.25, 0];
teal = RGBColor[0, 0.5, 0.5];
slategreen = RGBColor[0.2, 0.4, 0.4];
lightblue = RGBColor[0, 0.75, 1];
slateblue = RGBColor[0.167, 0.333, 0.5];
plum = RGBColor[0.5, 0, 0.5];
purple = RGBColor[0.333, 0, 0.667];
beige = RGBColor[0.8, 0.6, 0.6];
brown = RGBColor[0.5, 0.25, 0.25];
pinegreen = RGBColor[0.2, 0.7, 0.2];
gold = RGBColor[1, 155 / 255, 0];
fucshia = RGBColor[1, 0.2, 0.8];

solid = GrayLevel[0];
dot = Dashing[{0.005, 0.015}];
dash = Dashing[{0.03, 0.03}];
dashLong = Dashing[{0.05, 0.02, 0.05, 0.02}];
dashDot = Dashing[{0.05, 0.015, 0.01, 0.015}];
dashDotDot = Dashing[{0.05, 0.015, 0.01, 0.015, 0.01, 0.015}];
dashDashDot = Dashing[{0.05, 0.015, 0.05, 0.015, 0.015, 0.015}];
dashDashDotDot = Dashing[{0.05, 0.015, 0.05, 0.015, 0.015, 0.015, 0.015, 0.015}];

dashStyle = {solid, dash, dot, dashDot, dashDotDot, dashDashDot,
  dashDashDotDot, solid, dashLong, dashDashDot, dot, dashDotDot, dashDot};
dashStyleOrig = dashStyle;

```

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```

colorStyle := {red, purple, teal, blue, orange, plum, green,
  rose, slategreen, lightblue, beige, slateblue, brown, pinegreen, gold};
colorStyleOrig = colorStyle;

Choose line style.

lineStyle := If[kLineStyle == "dashing",
  Map[{#, thickness} &, dashStyle],
  Map[{#, thickness} &, colorStyle] ];

SetOptions[{Plot, LogPlot, LogLogPlot}, PlotStyle -> Table[lineStyle[[i, 1], {i, 1, 13}]]];

Point style

pointStyle := Map[{#, ptSize} &, colorStyle]

Set styles specific to various fusion reactions

dtStyle := lineStyle[[3]];
dheStyle := lineStyle[[1]];
dhep5Style := lineStyle[[5]];
dhe3t1Style := lineStyle[[4]];
ddStyle := lineStyle[[2]];
ddptStyle := lineStyle[[4]];
ddn3Style := lineStyle[[5]];
heheStyle := lineStyle[[4]];
pbStyle := lineStyle[[5]];
pb2Style := lineStyle[[10]];
pb3Style := lineStyle[[4]];
pb4Style := lineStyle[[6]];

```

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■ Fusion power and plasma packages

The fusion.m package imports plasma.m.

```
Needs["fusion`"]
```

■ Constants and units

```
Needs["Miscellaneous`PhysicalConstants`"];
Needs["Miscellaneous`Units`"]
```

```
 $\epsilon_0 = \text{VacuumPermittivity}$   

 $\frac{8.85419 \times 10^{-12} \text{ Ampere Second}}{\text{Meter Volt}}$ 
```

```
 $\mu_0 = \text{VacuumPermeability} // N$   

 $\frac{1.25664 \times 10^{-6} \text{ Second Volt}}{\text{Ampere Meter}}$ 
```

```
 $k_B = 1.6 \times 10^{-19} \text{ Joule / ElectronVolt}$   

 $\frac{1.6 \times 10^{-19} \text{ Joule}}{\text{ElectronVolt}}$ 
```

```
 $m_e = \text{ElectronMass}$   

 $9.10939 \times 10^{-31} \text{ Kilogram}$ 
```

```
 $m_p = \text{ProtonMass}$   

 $1.67262 \times 10^{-27} \text{ Kilogram}$ 
```

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■ Rules

```
rlJoule = {Joule → Kilogram Meter2 / Second2};
rlToJoule = {Joule → Kilogram Meter2 / Second2};
```

■ Functions

```
CheckZoneBoundaryMasses[m1_, m2_] :=
If[0.9 <  $\frac{m1}{m2}$  < 1.1,
Null,
Print[StyleForm["*** Zone boundary mass disconnect ***",
FontColor → red]]]
```

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Parameters based on Thio, et al, Current Trends in International Fusion Research 2nd Symposium and Thio, et al, AIAA-99-2703

Note: see reference section (Thio 1999a and Thio 1999b).

Using BUCKY routine zoner3 with varying zone masses and thicknesses within a given region.

■ Miscellaneous parameters

$f_{rep} = 40 \text{ Hertz};$

$r_w = 1.4 \text{ Meter};$

■ Target before impact

■ Plasma parameters

$B_t = 1.0 \text{ Tesla};$

$\mu_t = 2.5;$

$\gamma_t = 5 / 3;$

$n_{it} = 2.0 \times 10^{24} \text{ Meter}^{-3};$

$r_t = 0.05 \text{ Meter};$

$T_{et} = 2 \text{ ElectronVolt};$

$z_t = 1;$

Delayed definitions

$\epsilon_{et} := 1.5 T_{et};$

$\epsilon_{it} := 1.5 T_{it};$

$n_{et} := z_t n_{it};$

$T_{it} := T_{et};$

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$$m_t = \mu_t m_p n_{it} \frac{4}{3} \pi r_t^3;$$

Total, D, and T masses

$$\{1, 0.4, 0.6\} m_t \frac{10^6 \text{ mg}}{\text{Kilogram}}$$

{4.37892 mg, 1.75157 mg, 2.62735 mg}

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```

plasmaParams [ $\mu_t$ ,  $B_t$ [[1]],  $10^{-3} \epsilon_{et}$ [[1]],  $10^{-3} \epsilon_{it}$ [[1]],  $\gamma_t$ ,  $n_{it}$ [[1]],  $10^{-3} T_{et}$ [[1]],  $10^{-3} T_{it}$ [[1]],  $z_t$ ]
Electron density       $2. \times 10^{24}$       m-3
AlfvenVelocity        9749.26          m/s
beta                  3.2
CoulogEE              2.70049
CoulogIE              2.70049
CoulogII              2.35392
DebyeLength            $7.43135 \times 10^{-9}$  m
electronGyroFrequency  $1.76 \times 10^{11}$       s-1
electronGyroRadius      $4.75835 \times 10^{-6}$  m
electronPlasmaFrequency  $7.97616 \times 10^{13}$  s-1
electronThermalVelocity 840762.          m/s
ionAcousticVelocity    11319.6          m/s
ionGyroFrequency        $3.832 \times 10^7$       s-1
ionGyroRadius          0.000322441 m
ionPlasmaFrequency      $1.18064 \times 10^{12}$  s-1
ionThermalVelocity     12388.5          m/s
nueePerp           $1.26808 \times 10^{13}$  s-1
nueeSlow           $7.24618 \times 10^{12}$  s-1
nuieEnergy         $3.22053 \times 10^7$       s-1
nuieSlow           $1.23185 \times 10^9$       s-1
nuiiPerp           $1.46474 \times 10^{11}$  s-1

```

The next few statements define quantities listed in the table but also required for calculations.

```

 $\Lambda_{iit} = \text{CoulogII}[\mu_t, n_{it}[[1]], 10^{-3} T_{it}[[1]], z_t]$ 
2.35392

```

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```

 $\Lambda_{eet} = \text{CoulogEE}[n_{et}[[1]], 10^{-3} T_{et}[[1]]]$ 
2.70049

 $v_{st} = \text{ionAcousticVelocity}[\mu_t, \gamma_t, 10^{-3} T_{et}[[1]], z_t]$  Meter / Second;

 $\tau_{st} = \frac{r_t}{v_{st}}$ 
4.41712  $\times 10^{-6}$  Second

 $\tau_t = 0.0$  Second;
 $Q_t = 0.0$  Joule;

```

□ Mean free paths

```

 $\lambda_{i\text{perp}} = \frac{\text{ionThermalVelocity}[\mu_t, 10^{-3} T_{it}[[1]]]}{\text{nu}_{ii}\text{Perp}[\Lambda_{iit}, \mu_t, n_{it}[[1]], 10^{-3} T_{it}[[1]], z_t]}$ 
8.45785  $\times 10^{-8}$ 

 $\lambda_{e\text{perp}} = \frac{\text{electronThermalVelocity}[10^{-3} T_{et}[[1]]]}{\text{nu}_{ee}\text{Perp}[\Lambda_{eet}, n_{et}[[1]], 10^{-3} T_{et}[[1]]]}$ 
6.63018  $\times 10^{-8}$ 

```

■ BUCKY input

```

 $N_t = 25;$ 
 $\xi_t = 0.05;$ 
 $\Lambda_t = \text{NestList}[(1.0 + \xi_t) * \#, 1.0, N_t - 1];$ 
Zone masses (kg)

```

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```

m_t =  $\frac{\Lambda_t m_t[[1]]}{\text{Plus} @@ \Lambda_t}$ ;

{m_t[[1]], m_t[[N_t]]}

{9.17491 × 10-8, 2.959 × 10-7}

```

■ Plasma jet before impact

■ Plasma parameters

```

B_j = 0.0 Tesla;
γ_j = 5 / 3;
μ_j = 2.5;
η_j = 60;
N_j = 100;
T_ej = 26.7 ElectronVolt;
z_j = 1;

Delayed definitions

ε_ej := 1.5 T_ej;
ε_ij := 1.5 T_ij;
n_ej := z_j n_ij;
T_ij := T_ej;

m_j = Convert[0.2 Gram, Kilogram]

0.0002 Kilogram

M_j = m_j / η_j

3.33333 × 10-6 Kilogram

```

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```

ξ_j = x - 1.0 /. FindRoot[ $\frac{m_j[[1]] / m_t[[N_t]]}{\text{Plus} @@ \text{NestList}[(x * \#) \&, 1.0, N_j - 1]} == 1, \{x, 1\}]$ 

0.0315417

Λ_j = NestList[(1 + ξ_j) * #] &, 1.0, N_j - 1];

Zone masses (kg)

m_j =  $\frac{\Lambda_j m_j[[1]]}{\text{Plus} @@ \Lambda_j}$ ;

{m_j[[1]], m_j[[N_j]]} // ScientificForm

{2.959 × 10-7, 6.40229 × 10-6}

CheckZoneBoundaryMasses[m_t[[N_t]], m_j[[1]]]

r_j = r_t + 0.024 Meter;

n_ij = v /. Solve[m_j == μ_j m_p v  $\frac{4 \pi}{3} (r_j^3 - r_t^3)$ , v][[1]]

 $\frac{4.07472 \times 10^{25}}{\text{Meter}^3}$ 

v_sj = ionAcousticVelocity[μ_j, γ_j, 10-3 T_ej[[1]], z_j] Meter / Second

 $\frac{41359.2 \text{ Meter}}{\text{Second}}$ 

Perfect acoustic match condition:

rul = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten

{u1 →  $\frac{125231. \text{ Meter}}{\text{Second}}$ }

```

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```

vj = u1 / .rlul
125231. Meter
Second

{τj, τsj} =  $\frac{\mathbf{r}_j}{\{\mathbf{v}_j, \mathbf{v}_{sj}\}}$ 
{5.90908 × 10-7 Second, 1.7892 × 10-6 Second}

Qj = Convert[ $\frac{m_j}{2} v_j^2$ , Joule]
1.56828 × 106 Joule

Energy per jet
Qj / nj
26138. Joule

```

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```

plasmaParams[μj, Bj[[1]], 10-3 εej[[1]], 10-3 εij[[1]], γj, nij[[1]], 10-3 Tej[[1]], 10-3 Tij[[1]], zj]
Electron density      4.07472 × 1025 m-3
CoulogEE              4.89219
CoulogIE              4.89219
CoulogII              4.73407
DebyeLength           6.01554 × 10-9 m
electronPlasmaFrequency 3.60021 × 1014 s-1
electronThermalVelocity 3.07195 × 106 m/s
ionAcousticVelocity   41359.2 m/s
ionPlasmaFrequency    5.32909 × 1012 s-1
ionThermalVelocity    45264.7 m/s
nueePerp           9.5952 × 1012 s-1
nueeSlow           5.48297 × 1012 s-1
nuieEnergy         2.43688 × 107 s-1
nuieSlow           9.32105 × 108 s-1
nuiiPerp           1.23041 × 1011 s-1

```

The next two statements define quantities listed in the table but also required for calculations.

```

Λij = CoulogII[μj, nij[[1]], 10-3 Tij[[1]], zj]
4.73407

Λeej = CoulogEE[nej[[1]], 10-3 Tej[[1]]]
4.89219

```

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□ Mean free paths

$$\lambda_{i\text{perp}} = \frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} T_{ij}[[1]]]}{\text{nuiiPerp}[\Lambda_{ij}, \mu_j, n_{ij}[[1]], 10^{-3} T_{ij}[[1]], z_j]}$$

$$3.67884 \times 10^{-7}$$

$$\lambda_{e\text{perp}} = \frac{\text{electronThermalVelocity}[10^{-3} T_{ej}[[1]]]}{\text{nueePerp}[\Lambda_{ej}, n_{ej}[[1]], 10^{-3} T_{ej}[[1]]]}$$

$$3.20154 \times 10^{-7}$$

■ BUCKY input

Area of jets at interface between jets and target:

$$A_j = 4 \pi r_t^2$$

$$0.0314159 \text{ Meter}^2$$

Directed energy per ion or electron

$$\delta_{ej} = \text{Convert}\left[\frac{m_e}{2} v_j^2, \text{ElectronVolt}\right]$$

$$0.0445833 \text{ ElectronVolt}$$

$$\delta_{ij} = \text{Convert}\left[\frac{z_j m_p}{2} v_j^2, \text{ElectronVolt}\right]$$

$$81.8618 \text{ ElectronVolt}$$

$$Q_j = \text{Convert}\left[\frac{m_j}{2} v_j^2, \text{Joule}\right]$$

$$1.56828 \times 10^6 \text{ Joule}$$

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$$\Gamma_j = n_{ij} v_j$$

$$\frac{5.10282 \times 10^{30}}{\text{Meter}^2 \text{ Second}}$$

Mass and density of jet specified; finding outer jet radius

$$r_j = \text{Module}\left[\{s\},\right.$$

$$\left. s /. \text{Solve}\left[\frac{2 m_p}{m_j} n_{ij} * \frac{4 \pi}{3} (s^3 \text{ Meter}^3 - r_t^3) = 1.0, s\right]\right][[3]] \text{ Meter}$$

$$0.0780399 \text{ Meter}$$

$$\tau_j = \frac{r_j}{v_j}$$

$$6.23167 \times 10^{-7} \text{ Second}$$

$$\Delta_j = r_j - r_t$$

$$0.0280399 \text{ Meter}$$

■ Buffer

■ Buffer before impact

$$B_b = 0.0 \text{ Tesla};$$

$$\mu_b = 2.5;$$

$$\gamma_b = 5/3;$$

$$n_{ib} = 4.5 \times 10^{24} \text{ Meter}^{-3};$$

$$N_b = 24;$$

$$T_{eb} = T_{ej};$$

$$z_b = 1;$$

$$m_b = \text{Convert}[2.0 \text{ Gram}, \text{Kilogram}];$$

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Delayed definitions

```

εeb := 1.5 Teb;
εib := 1.5 Tib;
neb := zb nib;
Tib := Teb;

```

```

ξb = x - 1.0 /. FindRoot[
  
$$\frac{m_b[[1]] / m_j[[N_j]]}{\text{Plus} @@ \text{NestList}[(x * \#) \&, 1.0, N_b - 1]} == 1, \{x, 1\}]$$


```

0.185012

```

Λb = NestList[(1 + ξb) * #] &, 1.0, Nb - 1];

```

Zone masses (kg)

```

mb = 
$$\frac{\Lambda_b m_b[[1]]}{\text{Plus} @@ \Lambda_b};$$

{mb[[1]], mb[[Nb]]} // ScientificForm
{6.40229 × 10-6, 3.17656 × 10-4}

```

```

CheckZoneBoundaryMasses[mj[[Nj]], mb[[1]]]

```

```

ρb = μb mp nib
0.018817 Kilogram
Meter3

```

```

rb = s /. Solve[mb == 
$$\frac{4 \pi}{3} (s^3 - r_j^3) \rho_b, s] [[3]]$$


```

0.295676 Meter

```

vsb = ionAcousticVelocity[μb, γb, 10-3 Teb[[1]], zb] Meter / Second
41359.2 Meter
Second

```

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```

vb = vj
125231. Meter
Second

{τb, τsb} = 
$$\frac{r_b}{\{v_b, v_{sb}\}}; \text{ScientificForm}[\{\tau_b, \tau_{sb}\}]$$

{(2.36105 × 10-6) Second, (7.149 × 10-6) Second}

```

Flux of ions

```

Γb = nib vb // N
5.6354 × 1029
Meter2 Second

```

Buffer kinetic energy

```

Qb = Convert[
$$\frac{m_b}{2} v_b^2, \text{Joule}]$$


```

1.56828 × 10⁷ Joule

```

{μb, Bb[[1]], 10-3 εeb[[1]], 10-3 εib[[1]], γb, nib[[1]], 10-3 Teb[[1]], 10-3 Tib[[1]], zb}
{2.5, 0., 0.04005, 0.04005,  $\frac{5}{3}$ , 4.5 × 1024, 0.0267, 0.0267, 1}

```

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```

plasmaParams[ $\mu_b$ ,  $B_b$ [[1]],  $10^{-3} \epsilon_{eb}$ [[1]],  $10^{-3} \epsilon_{ib}$ [[1]],  $\gamma_b$ ,  $n_{ib}$ [[1]],  $10^{-3} T_{eb}$ [[1]],  $10^{-3} T_{ib}$ [[1]],  $z_b$ ]
Electron density       $4.5 \times 10^{24}$       m-3
CoulogEE              5.99385
CoulogIE              5.99385
CoulogII              5.83573
DebyeLength            $1.81016 \times 10^{-8}$  m
electronPlasmaFrequency  $1.19642 \times 10^{14}$  s-1
electronThermalVelocity  $3.07195 \times 10^6$  m/s
ionAcousticVelocity   41359.2 m/s
ionPlasmaFrequency     $1.77097 \times 10^{12}$  s-1
ionThermalVelocity    45264.7 m/s
nueePerp           $1.29829 \times 10^{12}$  s-1
nueeSlow           $7.41878 \times 10^{11}$  s-1
nuieEnergy         $3.29724 \times 10^6$  s-1
nuieSlow           $1.26119 \times 10^8$  s-1
nuiiPerp           $1.67503 \times 10^{10}$  s-1

```

■ Jet-target interface at impact

■ Basic parameters

Note: ion parameters are for the jets, while electron parameters are for the target, except for the electron-electron slowing down frequency, because the dominant physics should be the ion-electron slowing-down interaction.

★ *Note: exercise caution if using this approach!*

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```

plasmaParams[ $\mu_j$ ,  $B_t$ [[1]],  $10^{-3} \epsilon_{ej}$ [[1]],  $10^{-3} \epsilon_{ij}$ [[1]],  $\gamma_j$ ,  $n_{et}$ [[1]],  $n_{ij}$ [[1]],  $10^{-3} T_{et}$ [[1]],  $10^{-3} T_{ij}$ [[1]],  $z_j$ ]
Electron density       $2. \times 10^{24}$       m-3
AlfvenVelocity        2159.92 m/s
beta                  467.778
CoulogEE              2.70049
CoulogIE              2.70049
CoulogII              4.73407
DebyeLength            $7.43135 \times 10^{-9}$  m
electronGyroFrequency  $1.76 \times 10^{11}$  s-1
electronGyroRadius      $4.75835 \times 10^{-6}$  m
electronPlasmaFrequency  $7.97616 \times 10^{13}$  s-1
electronThermalVelocity 840762. m/s
ionAcousticVelocity    11319.6 m/s
ionGyroFrequency        $3.832 \times 10^7$  s-1
ionGyroRadius           0.00117812 m
ionPlasmaFrequency      $5.32909 \times 10^{12}$  s-1
ionThermalVelocity     45264.7 m/s
nueePerp           $1.26808 \times 10^{13}$  s-1
nueeSlow           $4.87594 \times 10^{14}$  s-1
nuieEnergy         $2.3746 \times 10^9$  s-1
nuieSlow           $1.23185 \times 10^9$  s-1
nuiiPerp           $1.23041 \times 10^{11}$  s-1

```

□ Coulomb logarithms

```

 $\Lambda_{eeI} = \text{CoulogEE}[n_{et}[[1]], 10^{-3} T_{et}[[1]]]$ 
2.70049

```

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$$\Lambda_{ieI} = \text{CoullogIE}[\mu_j, n_{et}[[1]], n_{ij}[[1]], 10^{-3} T_{et}[[1]], 10^{-3} \mathcal{E}_{ij}[[1]], z_j]$$

$$2.70049$$

$$\Lambda_{iiI} = \text{CoullogII}[\mu_j, \mu_t, n_{ij}[[1]], n_{it}[[1]], 10^{-3} \mathcal{E}_{ij}[[1]], 10^{-3} T_{it}[[1]], z_j, z_t]$$

$$5.54139$$

■ Mean free paths

Note: the relevant mean free paths for this part of the problem are for the interaction of the jet ions, with energy $\mathcal{E}_i$, impacting the target ions or electrons, with temperatures T_i and T_e .

$$\lambda_{iiperp} = \frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \mathcal{E}_{ij}[[1]]]}{\text{nuiiPerp}[\Lambda_{iiI}, \mu_j, n_{ij}[[1]], 10^{-3} \mathcal{E}_{ij}[[1]], z_j]}$$

$$2.95439 \times 10^{-6}$$

$$\lambda_{ies} = \frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \mathcal{E}_{ij}[[1]]]}{\text{nuieSlow}[10^{-3} \mathcal{E}_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]}$$

$$0.0000643408$$

$$\lambda_{ieE} = \frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \mathcal{E}_{ij}[[1]]]}{\text{nuieEnergy}[10^{-3} \mathcal{E}_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]}$$

$$0.0000333776$$

$$\lambda_{eeperp} = \frac{\text{electronThermalVelocity}[10^{-3} T_{et}[[1]]]}{\text{nuiePerp}[\Lambda_{eeI}, n_{et}[[1]], 10^{-3} T_{et}[[1]]]}$$

$$6.63018 \times 10^{-8}$$

■ Collapse time scale

Based on jet velocity

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$$\tau_{collapseJ} = \frac{r_t}{v_j}$$

$$3.99262 \times 10^{-7} \text{ Second}$$

Based on ion-acoustic velocity

$$\tau_{collapseS} = \frac{r_t}{v_{st}}$$

$$4.41712 \times 10^{-6} \text{ Second}$$

$$\tau_{collapse} = \text{Min}[\tau_{collapseJ}[[1]], \tau_{collapseS}[[1]]] \text{ Second}$$

$$3.99262 \times 10^{-7} \text{ Second}$$

■ Ion-electron interaction time scales

Slowing down

$$\tau_{ies} = \text{Second} / \text{nuieSlow}[10^{-3} \mathcal{E}_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]$$

$$8.11786 \times 10^{-10} \text{ Second}$$

Energy transfer

$$\tau_{ieE} = \text{Second} / \text{nuieEnergy}[10^{-3} \mathcal{E}_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]$$

$$4.21124 \times 10^{-10} \text{ Second}$$

$$\tau_{ie} = \text{Min}[\tau_{ies}[[1]], \tau_{ieE}[[1]]] \text{ Second}$$

$$4.21124 \times 10^{-10} \text{ Second}$$

■ Critical energy for ion slowing down

Jet ions into target

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```

Ecrit[1, {μj, μt}, {nij[[1]], nit[[1]]}, 10-3 Tet[[1]], {10-3 εij[[1]], 10-3 Tit[[1]]}, {zj, zt}] 103 ElectronVolt
113.339 ElectronVolt

```

■ Pressure

Jet pressure at initial jet-target impact

```

Pj0 = Convert[ $\frac{M_j v_j}{4 \pi r_t^2 \tau_{ie}}$ , Pascal]
3.15523 × 1010 Pascal

```

B-field pressure (Pa) at initial liner-target impact

```

PB0 = Convert[ $\frac{B_t^2}{2 \mu_0}$ , Pascal]
397887. Pascal

```

```

 $\frac{P_{j0}}{P_{B0}}$ 
79299.5

```

■ Velocity

Perfect acoustic match condition:

```

r1u1 = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
{u1 →  $\frac{125231. \text{Meter}}{\text{Second}}$ }

```

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■ Compressed target plasma

■ Basic parameters

```

Bc = 10.0 Tesla;
γc = 5 / 3;
μc = μt;
nic = 1.0 × 1025 Meter-3;
rc = 0.0025 Meter;
Tec = Convert[5 Kilo ElectronVolt, ElectronVolt];
zc = 1;

```

Delayed definitions

```

εec := 1.5 Tec;
εic := 1.5 Tic;
nec := zc nic;
Tic := Tec;

```

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```

plasmaParams[ $\mu_c$ ,  $B_c$ [[1]],  $10^{-3} \epsilon_{ec}$ [[1]],  $10^{-3} \epsilon_{ic}$ [[1]],  $\gamma_c$ ,  $n_{ic}$ [[1]],  $10^{-3} T_{ec}$ [[1]],  $10^{-3} T_{ic}$ [[1]],  $z_c$ ]
Electron density       $1. \times 10^{25}$       m-3
AlfvenVelocity        43600.          m/s
beta                  400.
CoulogEE              10.8271
CoulogIE              10.8271
CoulogII              13.2853
DebyeLength            $1.6617 \times 10^{-7}$  m
electronGyroFrequency  $1.76 \times 10^{12}$       s-1
electronGyroRadius    0.0000237918 m
electronPlasmaFrequency  $1.78352 \times 10^{14}$  s-1
electronThermalVelocity  $4.20381 \times 10^7$  m/s
ionAcousticVelocity   565980.          m/s
ionGyroFrequency        $3.832 \times 10^8$       s-1
ionGyroRadius          0.0016122 m
ionPlasmaFrequency     $2.64 \times 10^{12}$       s-1
ionThermalVelocity    619426.          m/s
nueePerp           $2.03366 \times 10^9$       s-1
nueeSlow           $1.16209 \times 10^9$       s-1
nuieEnergy        5164.84          s-1
nuieSlow          197555.          s-1
nuiiPerp           $3.30672 \times 10^7$       s-1

```

■ Mean free paths

```

 $\Lambda_{ic} = \text{CoulogII}[\mu_c, n_{ic}[[1]], 10^{-3} T_{ic}[[1]], z_c]$ 
13.2853

```

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```

 $\lambda_{i\text{perp}} = \frac{\text{ionThermalVelocity}[\mu_c, 10^{-3} T_{ic}[[1]]]}{\text{nu}_{ii}\text{Perp}[\Lambda_{ic}, \mu_c, n_{ic}[[1]], 10^{-3} T_{ic}[[1]], z_c]}$ 
0.0187323

 $\Lambda_{ec} = \text{CoulogEE}[n_{ec}[[1]], 10^{-3} T_{ec}[[1]]]$ 
10.8271

 $\lambda_{e\text{perp}} = \frac{\text{electronThermalVelocity}[10^{-3} T_{ec}[[1]]]}{\text{nu}_{ee}\text{Perp}[\Lambda_{ec}, n_{ec}[[1]], 10^{-3} T_{ec}[[1]]]}$ 
0.0206712

```

■ Critical energies for fusion-product slowing down

D-D alphas

```

Ecrit[1, {4,  $\mu_c$ }, { $10^{-4} n_{ic}$ [[1]],  $n_{ic}$ [[1]]},  $10^{-3} T_{ec}$ [[1]], { $10^{-3} T_{ic}$ [[1]],  $10^{-3} T_{ic}$ [[1]]}, {2,  $z_c$ }] keV
180.961 keV

```

D-D protons

```

Ecrit[1, {1,  $\mu_c$ }, { $10^{-4} n_{ic}$ [[1]],  $n_{ic}$ [[1]]},  $10^{-3} T_{ec}$ [[1]], { $10^{-3} T_{ic}$ [[1]],  $10^{-3} T_{ic}$ [[1]]}, {1,  $z_c$ }] keV
46.8453 keV

```

D-D tritons

```

Ecrit[1, {3,  $\mu_c$ }, { $10^{-4} n_{ic}$ [[1]],  $n_{ic}$ [[1]]},  $10^{-3} T_{ec}$ [[1]], { $10^{-3} T_{ic}$ [[1]],  $10^{-3} T_{ic}$ [[1]]}, {1,  $z_c$ }] keV
140.52 keV

```

D-³He alphas

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Ecrit[1, {4, μ_c }, { $10^{-4} n_{ic}[[1]$, $n_{ic}[[1]]$ }, $10^{-3} T_{ec}[[1]$, { $10^{-3} T_{ic}[[1]$, $10^{-3} T_{ic}[[1]]$ }, {2, z_c }] **keV**
 180.961 keV

■ Energy change

Increase in target energy

$\Delta \epsilon_t = 2 \pi k_B ((n_{ic} + n_{ec}) T_{ic} r_c^3 - (n_{it} + n_{et}) T_{it} r_t^3)$
 565.487 Joule

Maximum fraction of jet energy remaining at full compression

$f_{jc} = \frac{Q_j - \Delta \epsilon_t}{Q_j}$
 0.999639

■ Pressure

Maximum jet pressure (Pa) remaining at full compression (jet momentum scales as square root of jet energy)

$P_{jc} = \sqrt{f_{jc}} P_{j0}$
 3.15466×10^{10} Pascal

Target pressure (Pa) at full compression

Ptc = Convert[$n_{ic} T_{ic} + n_{ec} T_{ec}$, **Pascal**]
 1.60218×10^{10} Pascal

B-field pressure (Pa) at full compression

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Pbc = Convert[$\frac{B_c^2}{2 \mu_0}$, **Pascal**]

3.97887×10^7 Pascal

Pressure ratios

Pressure ratios

$\frac{P_{jc}}{\{P_{bc}, P_{tc}\}}$
 {792.852, 1.96898}

■ Summary

■ Setup print lists

$N_z = \{N_t, N_j, N_b\};$
 $m_{region} = \{m_t, m_j, m_b\};$
 $n_{region} = \{n_{it}, n_{ij}, n_{ib}\};$
 $Q_{region} = \{Q_t, Q_j, Q_b\};$
 $\tau_{region} = \{\tau_t, \tau_j, \tau_b\};$
 $\tau_{sregion} = \{\tau_{st}, \tau_{sj}, \tau_{sb}\};$
 $T_{region} = \{T_{et}, T_{ej}, T_{eb}\};$
 $v_{region} = \{\{0\}, v_j, v_b\};$
 $\xi_{region} = \{\xi_t, \xi_j, \xi_b\};$

Total number of zones

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Plus @@ N_z

149

■ Zone widths required for specified number of zones

$r_{\text{region}} = \{r_t, r_j, r_b\};$

$\Delta_{\text{region}} = -\text{Apply}[\text{Subtract}, \text{Partition}[\text{Prepend}[r_{\text{region}}, 0.0 \text{ Meter}], 2, 1], 1];$

■ Summary table

```
lsSummary = {Nz dummyunit, Sregion dummyunit, Δregion, rregion,
  103 mregion, nregion, Tregion, 10-3 vregion, 106 τregion, 106 τsregion, 10-6 Qregion};
TableForm[Map[#[[1]] &, lsSummary, {2}],
  TableHeadings → {{"zones", "zonfac", "Δregion (m)", "r (m)", "m (g)", "n (m-3)",
    "Te (eV)", "v (km/s)", "τ (μs)", "τs (μs)", "KE (MJ)"}, {"Target", "Jets", "Buffer"}}]
```

	Target	Jets	Buffer
zones	25	100	24
zonfac	0.05	0.0315417	0.185012
Δregion (m)	0.05	0.0280399	0.217637
r (m)	0.05	0.0780399	0.295676
m (g)	0.00437892	0.2	2.
n (m ⁻³)	$2. \times 10^{24}$	4.07472×10^{25}	4.5×10^{24}
T _e (eV)	2	26.7	26.7
v (km/s)	0	125.231	125.231
τ (μs)	0.	0.623167	2.36105
τ _s (μs)	4.41712	1.7892	7.149
KE (MJ)	0.	1.56828	15.6828

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Parameters based roughly on Francis Thio's 3/23/01 output case

The values in this section are based on notes from a discussion with Francis Thio on 3/23/01, and an output case from Francis' MATLAB program.

Case 020718.01 uses these parameters.

■ Input parameters

■ Miscellaneous parameters

$f_{\text{rep}} = 40 \text{ Hertz};$

$r_w = 1.4 \text{ Meter};$

■ Target before impact

$\mu_t = 2;$

$B_t = 1.0 \text{ Tesla};$

$\gamma_t = 5 / 3;$

$m_t = 3.56 \times 10^{-8} \text{ Kilogram};$

$r_t = 0.024 \text{ Meter};$

$T_{\text{et}} = 5.0 \text{ ElectronVolt};$

$z_t = 1;$

$$n_{\text{it}} = \frac{m_t}{\mu_t m_p \frac{4}{3} \pi r_t^3}$$

$$\frac{1.83781 \times 10^{23}}{\text{Meter}^3}$$

Delayed definitions

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```

 $\epsilon_{et} := 1.5 T_{et};$ 
 $\epsilon_{it} := 1.5 T_{it};$ 
 $n_{et} := z_t n_{it};$ 
 $T_{it} := T_{et};$ 

```

■ Plasma jets before impact

```

 $B_j = 0.0 \text{ Tesla};$ 
 $\gamma_j = 5 / 3;$ 
 $\mu_j = 2;$ 
 $N_j = 60;$ 
 $M_j = 3.21 \times 10^{-7} \text{ Kilogram};$ 
 $n_{ij} = 6.24 \times 10^{23} \text{ Meter}^{-3};$ 
 $T_{ej} = 2 \text{ ElectronVolt};$ 
 $v_j = \text{Convert}[327 \text{ Kilo Meter / Second}, \text{Meter / Second}];$ 
 $z_j = 1;$ 

```

Total jet mass

```

 $m_j = N_j M_j; \text{Convert}[m_j, \text{Gram}]$ 
0.01926 Gram

```

Delayed definitions

```

 $\epsilon_{ej} := 1.5 T_{ej};$ 
 $\epsilon_{ij} := 1.5 T_{ij};$ 
 $n_{ej} := z_j n_{ij};$ 
 $T_{ij} := T_{ej};$ 

```

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■ Buffer before impact

```

 $m_b = \text{Convert}[2.2 \text{ Gram}, \text{Kilogram}];$ 
 $\mu_b = 2;$ 
 $n_{ib} = 2.4 \times 10^{24} \text{ Meter}^{-3};$ 
 $v_b = \text{Convert}[125 \text{ Kilo Meter / Second}, \text{Meter / Second}];$ 

```

■ Compressed plasma

```

 $B_c = 10.0 \text{ Tesla};$ 
 $\gamma_c = 5 / 3;$ 
 $\mu_c = \mu_t;$ 
 $n_{ic} = 1.0 \times 10^{25} \text{ Meter}^{-3};$ 
 $r_c = 0.008 \text{ Meter};$ 
 $T_{ec} = \text{Convert}[10 \text{ Kilo ElectronVolt}, \text{ElectronVolt}];$ 
 $z_c = 1;$ 

```

Delayed definitions

```

 $\epsilon_{ec} := 1.5 T_{ec};$ 
 $\epsilon_{ic} := 1.5 T_{ic};$ 
 $n_{ec} := z_c n_{ic};$ 
 $T_{ic} := T_{ec};$ 

```

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■ Target before impact

plasmaParams $[\mu_t, B_t[[1]], 10^{-3} \epsilon_{et}[[1]], 10^{-3} \epsilon_{it}[[1]], \gamma_t, n_{it}[[1]], 10^{-3} T_{et}[[1]], 10^{-3} T_{it}[[1]], z_t]$

Electron density	1.83781×10^{23}	m^{-3}
AlfvenVelocity	35957.7	m/s
beta	0.735122	
CoulogEE	5.26851	
CoulogIE	5.26851	
CoulogII	4.92194	
DebyeLength	3.87617×10^{-8}	m
electronGyroFrequency	1.76×10^{11}	s^{-1}
electronGyroRadius	7.52362×10^{-6}	m
electronPlasmaFrequency	2.41785×10^{13}	s^{-1}
electronThermalVelocity	1.32936×10^6	m/s
ionAcousticVelocity	20010.4	m/s
ionGyroFrequency	4.79×10^7	s^{-1}
ionGyroRadius	0.000456	m
ionPlasmaFrequency	4.00137×10^{11}	s^{-1}
ionThermalVelocity	21900.	m/s
nueePerp	5.75111×10^{11}	s^{-1}
nueeSlow	3.28635×10^{11}	s^{-1}
nuieEnergy	1.82575×10^6	s^{-1}
nuieSlow	6.98349×10^7	s^{-1}
nuiiPerp	7.96009×10^9	s^{-1}

$v_{st} = \text{ionAcousticVelocity}[\mu_t, \gamma_t, 10^{-3} T_{et}[[1]], z_t]$ Meter / Second
 $\frac{20010.4 \text{ Meter}}{\text{Second}}$

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□ Coulomb logarithms

$\Lambda_{iit} = \text{CoulogII}[\mu_t, n_{it}[[1]], 10^{-3} T_{it}[[1]], z_t]$

4.92194

$\Lambda_{eet} = \text{CoulogEE}[n_{et}[[1]], 10^{-3} T_{et}[[1]]]$

5.26851

■ Mean free paths

$\lambda_{iiperp} = \frac{\text{ionThermalVelocity}[\mu_t, 10^{-3} T_{it}[[1]]]}{\text{nuiiPerp}[\Lambda_{iit}, \mu_t, n_{it}[[1]], 10^{-3} T_{it}[[1]], z_t]}$

2.75122×10^{-6}

$\lambda_{eeperp} = \frac{\text{electronThermalVelocity}[10^{-3} T_{et}[[1]]]}{\text{nueePerp}[\Lambda_{eet}, n_{et}[[1]], 10^{-3} T_{et}[[1]]]}$

2.31149×10^{-6}

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■ Plasma jet before impact

■ Basic parameters

`plasmaParams`[μ_j , B_j [[1]], $10^{-3} \epsilon_{ej}$ [[1]], $10^{-3} \epsilon_{ij}$ [[1]], γ_j , n_{ij} [[1]], $10^{-3} T_{ej}$ [[1]], $10^{-3} T_{ij}$ [[1]], z_j]

Electron density	6.24×10^{23}	m^{-3}
CoulogEE	3.28287	
CoulogIE	3.28287	
CoulogII	2.9363	
DebyeLength	1.33043×10^{-8}	m
electronPlasmaFrequency	4.45524×10^{13}	s^{-1}
electronThermalVelocity	840762.	m/s
ionAcousticVelocity	12655.7	m/s
ionPlasmaFrequency	7.37312×10^{11}	s^{-1}
ionThermalVelocity	13850.8	m/s
nueePerp	4.80964×10^{12}	s^{-1}
nueeSlow	2.74837×10^{12}	s^{-1}
nuieEnergy	1.52687×10^7	s^{-1}
nuieSlow	5.84028×10^8	s^{-1}
nuiiPerp	6.37349×10^{10}	s^{-1}

□ Coulomb logarithms

$\Lambda_{ij} = \text{CoulogII}[\mu_j, n_{ij}[[1]], 10^{-3} T_{ij}[[1]], z_j]$

2.9363

$\Lambda_{eej} = \text{CoulogEE}[n_{ej}[[1]], 10^{-3} T_{ej}[[1]]]$

3.28287

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■ Mean free paths

$\lambda_{iiperp} = \frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} T_{ij}[[1]]]}{\text{nuiiPerp}[\Lambda_{ij}, \mu_j, n_{ij}[[1]], 10^{-3} T_{ij}[[1]], z_j]}$

2.17319×10^{-7}

$\lambda_{eeperp} = \frac{\text{electronThermalVelocity}[10^{-3} T_{ej}[[1]]]}{\text{nueePerp}[\Lambda_{eej}, n_{ej}[[1]], 10^{-3} T_{ej}[[1]]]}$

1.74808×10^{-7}

■ Jet parameters

Area of jets at interface between jets and target:

$A_j = 4 \pi r_t^2$

0.00723823 Meter²

Directed energy per ion or electron

$\delta_{ej} = \text{Convert}\left[\frac{m_e}{2} v_j^2, \text{ElectronVolt}\right]$

0.303979 ElectronVolt

$\delta_{ij} = \text{Convert}\left[\frac{z_j m_p}{2} v_j^2, \text{ElectronVolt}\right]$

558.153 ElectronVolt

$Q_j = \text{Convert}\left[\frac{m_j}{2} v_j^2, \text{Joule}\right]$

1.02973×10^6 Joule

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```


$$\Gamma_j = n_{ij} v_j$$


$$\frac{2.04048 \times 10^{29}}{\text{Meter}^2 \text{ Second}}$$


$$r_j = \text{Module}[\{s\}, s /. \text{Solve}[\frac{2 m_p}{m_j} n_{ij} * \frac{4 \pi}{3} (s^3 \text{Meter}^3 - r_t^3) == 1.0, s]] \llbracket 3 \rrbracket \text{Meter}$$

0.130384 Meter

$$\tau_j = \frac{r_j}{v_j}$$


$$3.98728 \times 10^{-7} \text{ Second}$$


$$\Delta_j = r_j - r_t$$

0.106384 Meter

```

■ Buffer

```


$$\rho_b = \mu_b m_p n_{ib}$$


$$\frac{0.00802859 \text{ Kilogram}}{\text{Meter}^3}$$


$$r_{lRb} = \text{Module}[\{s\}, s /. \text{Solve}[\rho_b == \frac{m_b}{\frac{4 \pi}{3} (s^3 - r_j^3)}, s]] \llbracket 3 \rrbracket$$

0.407432 Meter

```

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■ Jet-target interface at impact

■ Basic parameters

Note: ion parameters are for the jets, while electron parameters are for the target, except for the electron-electron slowing down frequency, because the dominant physics should be the ion-electron slowing-down interaction.

★ Note: exercise caution if using this approach!

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```

plasmaParams[μj, Bt[[1]], 10-3 εej[[1]], 10-3 εij[[1]], γj, net[[1]], nij[[1]], 10-3 Tet[[1]], 10-3 Tij[[1]], zj]
Electron density      1.83781 × 1023 m-3
AlfvenVelocity        19514.1 m/s
beta                  1.7472
CoulogEE              5.26851
CoulogIE              5.26851
CoulogII              2.9363
DebyeLength           3.87617 × 10-8 m
electronGyroFrequency 1.76 × 1011 s-1
electronGyroRadius     7.52362 × 10-6 m
electronPlasmaFrequency 2.41785 × 1013 s-1
electronThermalVelocity 1.32936 × 106 m/s
ionAcousticVelocity    20010.4 m/s
ionGyroFrequency       4.79 × 107 s-1
ionGyroRadius          0.0002884 m
ionPlasmaFrequency     7.37312 × 1011 s-1
ionThermalVelocity     13850.8 m/s
nueePerp          5.75111 × 1011 s-1
nueeSlow          8.10832 × 1012 s-1
nuieEnergy        1.37818 × 108 s-1
nuieSlow          6.98349 × 107 s-1
nuiiPerp          6.37349 × 1010 s-1

```

□ Coulomb logarithms

```

ΛeeI = CoulogEE[net[[1]], 10-3 Tet[[1]]]
5.26851

```

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```

ΛieI = CoulogIE[μj, net[[1]], nij[[1]], 10-3 Tet[[1]], 10-3 εij[[1]], zj]
5.26851

ΛiiI = CoulogII[μj, μt, nij[[1]], nit[[1]], 10-3 εij[[1]], 10-3 Tit[[1]], zj, zt]
9.28449

```

■ Mean free paths

Note: the relevant mean free paths for this part of the problem are for the interaction of the jet ions, with energy $\mathcal{E}_i$, impacting the target ions or electrons, with temperatures T_i and T_e .

```

λiiPerp =  $\frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \epsilon_{ij}[[1]]]}{\text{nuIIPerp}[\Lambda_{iiI}, \mu_j, n_{ij}[[1]], 10^{-3} \epsilon_{ij}[[1]], z_j]}$ 
0.00535284

λieS =  $\frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \epsilon_{ij}[[1]]]}{\text{nuieSlow}[10^{-3} \epsilon_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]}$ 
0.00331332

λieE =  $\frac{\text{ionThermalVelocity}[\mu_j, 10^{-3} \epsilon_{ij}[[1]]]}{\text{nuieEnergy}[10^{-3} \epsilon_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]}$ 
0.00167892

λeePerp =  $\frac{\text{electronThermalVelocity}[10^{-3} T_{et}[[1]]]}{\text{nueePerp}[\Lambda_{eeI}, n_{et}[[1]], 10^{-3} T_{et}[[1]]]}$ 
2.31149 × 10-6

```

■ Collapse time scale

Based on jet velocity

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$$\tau_{\text{collapseJ}} = \frac{r_t}{v_j}$$

$$7.33945 \times 10^{-8} \text{ Second}$$

Based on ion-acoustic velocity

$$\tau_{\text{collapseS}} = \frac{r_t}{v_{st}}$$

$$1.19938 \times 10^{-6} \text{ Second}$$

$$\tau_{\text{collapse}} = \text{Min}[\tau_{\text{collapseJ}}[[1]], \tau_{\text{collapseS}}[[1]]] \text{ Second}$$

$$7.33945 \times 10^{-8} \text{ Second}$$

■ Ion-electron interaction time scales

Slowing down

$$\tau_{ieS} = \text{Second} / \text{nuieSlow}[10^{-3} \varepsilon_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]$$

$$1.43195 \times 10^{-8} \text{ Second}$$

Energy transfer

$$\tau_{ieE} = \text{Second} / \text{nuieEnergy}[10^{-3} \varepsilon_{ij}[[1]], \Lambda_{ieI}, \mu_j, n_{et}[[1]], 10^{-3} T_{et}[[1]], z_j]$$

$$7.25597 \times 10^{-9} \text{ Second}$$

$$\tau_{ie} = \text{Min}[\tau_{ieS}[[1]], \tau_{ieE}[[1]]] \text{ Second}$$

$$7.25597 \times 10^{-9} \text{ Second}$$

■ Critical energy for ion slowing down

Jet ions into target

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$$\text{Ecrit}[1, \{\mu_j, \mu_t\}, \{n_{ij}[[1]], n_{it}[[1]]\}, 10^{-3} T_{et}[[1]], \{10^{-3} \varepsilon_{ij}[[1]], 10^{-3} T_{it}[[1]]\}, \{z_j, z_t\}] 10^3 \text{ ElectronVolt}$$

$$150.474 \text{ ElectronVolt}$$

■ Pressure

Jet pressure at initial jet-target impact

$$P_{j0} = \text{Convert}\left[\frac{\mathcal{M}_j v_j}{4 \pi r_t^2 \tau_{ie}}, \text{Pascal}\right]$$

$$1.9986 \times 10^9 \text{ Pascal}$$

B-field pressure (Pa) at initial liner-target impact

$$P_{B0} = \text{Convert}\left[\frac{B_t^2}{2 \mu_0}, \text{Pascal}\right]$$

$$397887. \text{ Pascal}$$

$$\frac{P_{j0}}{P_{B0}}$$

$$5023.02$$

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■ Compressed target plasma

■ Basic parameters

plasmaParams $[\mu_c, B_c[[1]], 10^{-3} \epsilon_{ec}[[1]], 10^{-3} \epsilon_{ic}[[1]], \gamma_c, n_{ic}[[1]], 10^{-3} T_{ec}[[1]], 10^{-3} T_{ic}[[1]], z_c]$

Electron density	$1. \times 10^{25}$	m^{-3}
AlfvenVelocity	48746.3	m/s
beta	800.	
CoulogEE	11.5203	
CoulogIE	11.5203	
CoulogII	14.325	
DebyeLength	2.35×10^{-7}	m
electronGyroFrequency	1.76×10^{12}	s^{-1}
electronGyroRadius	0.0000336466	m
electronPlasmaFrequency	1.78352×10^{14}	s^{-1}
electronThermalVelocity	5.94508×10^7	m/s
ionAcousticVelocity	894893.	m/s
ionGyroFrequency	4.79×10^8	s^{-1}
ionGyroRadius	0.00203929	m
ionPlasmaFrequency	2.95161×10^{12}	s^{-1}
ionThermalVelocity	979398.	m/s
nueePerp	7.65036×10^8	s^{-1}
nueeSlow	4.37164×10^8	s^{-1}
nuieEnergy	2428.69	s^{-1}
nuieSlow	92897.3	s^{-1}
nuiiPerp	1.40939×10^7	s^{-1}

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■ Mean free paths

$\Lambda_{iic} = \text{CoulogII}[\mu_c, n_{ic}[[1]], 10^{-3} T_{ic}[[1]], z_c]$

14.325

$\lambda_{iiperp} = \frac{\text{ionThermalVelocity}[\mu_c, 10^{-3} T_{ic}[[1]]]}{\text{nuiiPerp}[\Lambda_{iic}, \mu_c, n_{ic}[[1]], 10^{-3} T_{ic}[[1]], z_c]}$

0.0694908

$\Lambda_{eec} = \text{CoulogEE}[n_{ec}[[1]], 10^{-3} T_{ec}[[1]]]$

11.5203

$\lambda_{eeperp} = \frac{\text{electronThermalVelocity}[10^{-3} T_{ec}[[1]]]}{\text{nueePerp}[\Lambda_{eec}, n_{ec}[[1]], 10^{-3} T_{ec}[[1]]]}$

0.0777098

■ Critical energies for fusion-product slowing down

D-D alphas

Ecrit $[1, \{4, \mu_c\}, \{10^{-4} n_{ic}[[1]], n_{ic}[[1]]\}, 10^{-3} T_{ec}[[1]], \{10^{-3} T_{ic}[[1]], 10^{-3} T_{ic}[[1]]\}, \{2, z_c\}]$ keV

424.251 keV

D-D protons

Ecrit $[1, \{1, \mu_c\}, \{10^{-4} n_{ic}[[1]], n_{ic}[[1]]\}, 10^{-3} T_{ec}[[1]], \{10^{-3} T_{ic}[[1]], 10^{-3} T_{ic}[[1]]\}, \{1, z_c\}]$ keV

109.549 keV

D-D tritons

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$\mathbf{Ecrit}[1, \{3, \mu_c\}, \{10^{-4} n_{ic}[[1], n_{ic}[[1]]\}, 10^{-3} T_{ec}[[1], \{10^{-3} T_{ic}[[1], 10^{-3} T_{ic}[[1]]\}, \{1, z_c\}]] \text{ keV}$
 328.617 keV

■ Energy change

Increase in target energy

$\Delta \epsilon_t = 2 \pi k_B ((n_{ic} + n_{ec}) T_{ic} r_c^3 - (n_{it} + n_{et}) T_{it} r_t^3)$
 102918. Joule

Maximum fraction of jet energy remaining at full compression

$f_{jc} = \frac{Q_j - \Delta \epsilon_t}{Q_j}$
 0.900053

■ Pressure

Maximum jet pressure (Pa) remaining at full compression (jet momentum scales as square root of jet energy)

$P_{jc} = \sqrt{f_{jc}} P_{j0}$
 $1.89609 \times 10^9 \text{ Pascal}$

Target pressure (Pa) at full compression

$P_{tc} = \text{Convert}[n_{ic} T_{ic} + n_{ec} T_{ec}, \text{Pascal}]$
 $3.20435 \times 10^{10} \text{ Pascal}$

B-field pressure (Pa) at full compression

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$P_{Bc} = \text{Convert}\left[\frac{B_c^2}{2 \mu_0}, \text{Pascal}\right]$
 $3.97887 \times 10^7 \text{ Pascal}$

Pressure ratios

Pressure ratios

$\frac{P_{jc}}{\{P_{Bc}, P_{tc}\}}$
 $\{47.6539, 0.0591723\}$

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BUCKY input parameters for selected cases

■ Case 030201.03

Based on Thio, et al, *Current Trends (1999)*. Using BUCKY routine zoner3 with varying zone masses and thicknesses within a given region.

■ Target

```

μt = 2.5;
γt = 5 / 3;
nit = 1.87 × 1024 Meter-3;
Tet = 2 ElectronVolt;
zt = 1;
ξt = 0.05;

rt = 0.05 Meter
0.05 Meter

```

```

mt = μt mp nit  $\frac{4 \pi}{3} r_t^3$ 
4.09429 × 10-6 Kilogram

```

Total, D, and T masses

```

Convert[mt, Milli Gram] {1, 0.4, 0.6}
{4.09429 GramMilli, 1.63771 GramMilli, 2.45657 GramMilli}

Nt = 25;

```

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```

ξt = 1.0 + ξt
1.05

```

```

Λt = NestList[(ξt * #) &, 1.0, Nt - 1]
{1., 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071,
 1.47746, 1.55133, 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893,
 2.18287, 2.29202, 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251}

```

Zone masses (kg)

```

mt =  $\frac{\Lambda_t m_t[[1]]}{\text{Plus}@@\Lambda_t}$ 
{8.57854 × 10-8, 9.00747 × 10-8, 9.45784 × 10-8, 9.93073 × 10-8, 1.04273 × 10-7, 1.09486 × 10-7, 1.14961 × 10-7,
 1.20709 × 10-7, 1.26744 × 10-7, 1.33081 × 10-7, 1.39735 × 10-7, 1.46722 × 10-7, 1.54058 × 10-7,
 1.61761 × 10-7, 1.69849 × 10-7, 1.78342 × 10-7, 1.87259 × 10-7, 1.96622 × 10-7, 2.06453 × 10-7,
 2.16775 × 10-7, 2.27614 × 10-7, 2.38995 × 10-7, 2.50945 × 10-7, 2.63492 × 10-7, 2.76666 × 10-7}

```

```

vst = ionAcousticVelocity[μt, γt, 10-3 Tet[[1]], zt] Meter / Second
11319.6 Meter
Second

```

```

 $\frac{r_t}{v_{st}}$ 
4.41712 × 10-6 Second

```

■ Jet

```

γj = 5 / 3;
μt = 2.5;
Tej = 12 ElectronVolt;
zj = 1;

```

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```

η = 60;

mj = Convert[0.4 Gram, Kilogram]
0.0004 Kilogram

Mj = mj / ηj
0.0004 Kilogram
60j

Nj = 100;

ξj = x /. FindRoot[ $\frac{m_j[[1]]}{m_t[N_t]]$  == 1, {x, 1}]
Plus @@ NestList[(x * #) &, 1.0, Nj - 1]
1.04212

ξj = ξj - 1.0
0.0421166

Λj = NestList[(ξj * #) &, 1.0, Nj - 1];

Zone masses (kg)

mj =  $\frac{\Lambda_j m_j[[1]]}{\text{Plus} @@ \Lambda_j}$ ;
{mj[[1]], mj[[Nj]]} // ScientificForm
{2.76666 × 10-7, 1.64313 × 10-5}

rj = rt + 0.024 Meter;

```

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```

nij = v /. Solve[mj == 2.5 mp v  $\frac{4 \pi}{3}$  (rj3 - rt3), v] [[1]]
 $\frac{8.14944 \times 10^{25}}{\text{Meter}^3}$ 

vsj = ionAcousticVelocity[μj, γj, 10-3 Tej[[1]], zj] Meter / Second
 $\frac{31000. \text{Meter}}{\text{Second}}$ 

Perfect acoustic match condition:

rlul = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
{u1 →  $\frac{137281. \text{Meter}}{\text{Second}}$ }

vj = u1 /. rlul
 $\frac{137281. \text{Meter}}{\text{Second}}$ 
 $\frac{r_j}{\{v_j, v_{sj}\}}$ 
{5.3904 × 10-7 Second, 2.3871 × 10-6 Second}

Qj = Convert[ $\frac{m_j}{2}$  vj2, Joule]
3.76923 × 106 Joule

Energy per jet

Qj / ηj
 $\frac{3.76923 \times 10^6 \text{ Joule}}{60_j}$ 

```

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■ Buffer

```

 $\mu_b = 2.5;$ 
 $\gamma_b = 5/3;$ 
 $T_{eb} = T_{ej};$ 
 $z_b = 1;$ 

 $n_{ib} = 4.5 \times 10^{24} \text{ Meter}^{-3};$ 

 $m_b = \text{Convert}[1.8 \text{ Gram}, \text{Kilogram}];$ 

 $N_b = 24;$ 

 $\xi_b = x /. \text{FindRoot}\left[\frac{m_b[[1]] / m_j[[N_j]]}{\text{Plus}@@\text{NestList}[(x * \#) \&, 1.0, N_b - 1]} == 1, \{x, 1\}\right]$ 
1.11481

 $\xi_b = \xi_b - 1.0$ 
0.114808

 $\Lambda_b = \text{NestList}[(\xi_b * \#) \&, 1.0, N_b - 1];$ 

Zone masses (kg)
 $m_b = \frac{\Lambda_b m_b[[1]]}{\text{Plus}@@\Lambda_b};$ 
 $\{m_b[[1]], m_b[[N_b]]\} // \text{ScientificForm}$ 
 $\{1.64313 \times 10^{-5}, 2.00111 \times 10^{-4}\}$ 

 $\rho_b = 2.5 m_p n_{ib}$ 
 $\frac{0.018817 \text{ Kilogram}}{\text{Meter}^3}$ 

```

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```

 $r_b = s /. \text{Solve}\left[m_b == \frac{4 \pi}{3} (s^3 - r_b^3) \rho_b, s\right][[3]]$ 
0.28538 Meter

 $v_{sb} = \text{ionAcousticVelocity}[\mu_b, \gamma_b, 10^{-3} T_{eb}[[1]], z_b] \text{ Meter / Second}$ 
 $\frac{27727.2 \text{ Meter}}{\text{Second}}$ 

 $v_b = \text{Convert}[100 \text{ Kilo Meter / Second}, \text{Meter / Second}];$ 

 $\frac{r_b}{\{v_b, v_{sb}\}} // \text{ScientificForm}$ 
 $\{(2.8538 \times 10^{-6}) \text{ Second}, (1.02924 \times 10^{-5}) \text{ Second}\}$ 

Flux of ions
 $\Gamma_b = n_{ib} v_b // N$ 
 $\frac{4.5 \times 10^{29}}{\text{Meter}^2 \text{ Second}}$ 

Buffer kinetic energy
 $Q_b = \text{Convert}\left[\frac{m_b}{2} v_b^2, \text{Joule}\right]$ 
 $9. \times 10^6 \text{ Joule}$ 

```

■ Zone widths required for specified number of zones

```

 $N_z = \{N_t, N_j, N_b\}$ 
 $\{25, 100, 24\}$ 

```

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```

mregion = {mt, mj, mb}; Map[Convert[#, Gram] &, mregion]
{0.00409429 Gram, 0.4 Gram, 1.8 Gram}

rregion = {rt, rj, rb}
{0.05 Meter, 0.074 Meter, 0.28538 Meter}

Δregion = -Apply[Subtract, Partition[Prepend[rregion, 0], 2, 1], 1]
{0.05 Meter, 0.024 Meter, 0.21138 Meter}

Total number of zones

Plus @@ Nz

149

```

■ Summary

```

lsSummary = {{ξt, ξj, ξb} dummyunit, Δregion,
  rregion, 103 {mt, mj, mb}, {nit, nij, nib}, {Tet, Tej, Teb}, 10-3 {{0}, vj, vb}};
TableForm[Map[#[[1]] &, lsSummary, {2}],
  TableHeadings → {"zonfac", "Δregion (m)", "r (m)", "m (g)", "n (m-3)", "Te (eV)", "v (km/s)"},
  {"Target", "Jets", "Buffer"}}]

```

	Target	Jets	Buffer
zonfac	0.05	0.0421166	0.114808
Δregion (m)	0.05	0.024	0.21138
r (m)	0.05	0.074	0.28538
m (g)	0.00409429	0.4	1.8
n (m ⁻³)	1.87 × 10 ²⁴	8.14944 × 10 ²⁵	4.5 × 10 ²⁴
T _e (eV)	2	12	12
v (km/s)	0	137.281	100

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■ Case 030313.01 (also 030201.05)

Based on Thio, et al, *Current Trends (1999)* with $v_{jet}=398$ km/s for "perfect acoustic matching."
Using BUCKY routine zoner3 with varying zone masses and thicknesses within a given region.

■ Target

```

μt = 2.5;
γt = 5 / 3;
nit = 0.43 × 1024 Meter-3;
Tet = 2 ElectronVolt;
zt = 1;
ξt = 0.05;

rt = 0.05 Meter
0.05 Meter

mt = μt mp nit  $\frac{4 \pi}{3} r_t^3$ 
9.41467 × 10-7 Kilogram

Total, D, and T masses
Convert[mt, Milli Gram] {1, 0.4, 0.6}
{0.941467 Gram Milli, 0.376587 Gram Milli, 0.56488 Gram Milli}

Nt = 25;
ξt = 1.0 + ξt
1.05

```

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$\Lambda_t = \text{NestList}[(\xi_t * \#) \&, 1.0, N_t - 1]$

{1., 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071,
1.47746, 1.55133, 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893,
2.18287, 2.29202, 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251}

Zone masses (kg)

$$m_t = \frac{\Lambda_t m_t[[1]]}{\text{Plus} @@ \Lambda_t}$$

{1.97261 × 10⁻⁸, 2.07124 × 10⁻⁸, 2.1748 × 10⁻⁸, 2.28354 × 10⁻⁸, 2.39771 × 10⁻⁸, 2.5176 × 10⁻⁸, 2.64348 × 10⁻⁸,
2.77565 × 10⁻⁸, 2.91444 × 10⁻⁸, 3.06016 × 10⁻⁸, 3.21317 × 10⁻⁸, 3.37382 × 10⁻⁸, 3.54252 × 10⁻⁸,
3.71964 × 10⁻⁸, 3.90562 × 10⁻⁸, 4.1009 × 10⁻⁸, 4.30595 × 10⁻⁸, 4.52125 × 10⁻⁸, 4.74731 × 10⁻⁸,
4.98467 × 10⁻⁸, 5.23391 × 10⁻⁸, 5.4956 × 10⁻⁸, 5.77038 × 10⁻⁸, 6.0589 × 10⁻⁸, 6.36185 × 10⁻⁸}

$v_{st} = \text{ionAcousticVelocity}[\mu_t, \gamma_t, 10^{-3} T_{et}[[1]], z_t] \text{ Meter / Second}$
11319.6 Meter
Second

$$\tau_{st} = \frac{r_t}{v_{st}}$$

4.41712 × 10⁻⁶ Second

$\tau_t = 0.0 \text{ Second};$

$Q_t = 0.0 \text{ Joule};$

■ Jet

$\gamma_j = 5 / 3;$

$\mu_j = 2.5;$

$T_{ej} = 29 \text{ ElectronVolt};$

$z_j = 1;$

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$n = 60;$

$m_j = \text{Convert}[0.4 \text{ Gram}, \text{Kilogram}]$

0.0004 Kilogram

$M_j = m_j / n_j$

$\frac{0.0004 \text{ Kilogram}}{60_j}$

$N_j = 100;$

$\xi_j = x /. \text{FindRoot}\left[\frac{m_j[[1]] / m_t[[N_t]]}{\text{Plus} @@ \text{NestList}[(x * \#) \&, 1.0, N_j - 1]} = 1, \{x, 1\}\right]$

1.0614

$\xi_j = \xi_j - 1.0$

0.0613959

$\Lambda_j = \text{NestList}[(\xi_j * \#) \&, 1.0, N_j - 1];$

Zone masses (kg)

$$m_j = \frac{\Lambda_j m_j[[1]]}{\text{Plus} @@ \Lambda_j};$$

{ $m_j[[1]]$, $m_j[[N_j]]$ } // ScientificForm

{6.36185 × 10⁻⁸, 2.31977 × 10⁻⁵}

$r_j = r_t + 0.024 \text{ Meter};$

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```

nij = v /. Solve[mj == 2.5 mp v  $\frac{4 \pi}{3}$  (rj3 - rt3), v][[1]]

$$\frac{8.14944 \times 10^{25}}{\text{Meter}^3}$$

vsj = ionAcousticVelocity[μj, γj, 10-3 Tej[[1]], zj] Meter / Second

$$\frac{43103.8 \text{ Meter}}{\text{Second}}$$

Perfect acoustic match condition:
rlul = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
{u1 →  $\frac{398062. \text{ Meter}}{\text{Second}}$ }
vj = u1 /. rlul

$$\frac{398062. \text{ Meter}}{\text{Second}}$$

{τj, τsj} =  $\frac{\mathbf{r}_j}{\{\mathbf{v}_j, \mathbf{v}_{sj}\}}$ 
{1.85901 × 10-7 Second, 1.71679 × 10-6 Second}
Qj = Convert[ $\frac{m_j}{2} v_j^2$ , Joule]
3.16907 × 107 Joule
Energy per jet
Qj / nj

$$\frac{3.16907 \times 10^7 \text{ Joule}}{60_j}$$


```

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■ Buffer

```

μb = 2.5;
γb = 5 / 3;
Teb = Tej;
zb = 1;

nib = 4.5 × 1024 Meter-3;
mb = Convert[1.8 Gram, Kilogram];
Nb = 24;

ξb = x /. FindRoot[ $\frac{m_b[[1]] / m_j[[N_j]]}{\text{Plus}@@\text{NestList}[(x * \#) \&, 1.0, N_b - 1]} == 1, \{x, 1\}]$ 
1.09074

ξb = ξb - 1.0
0.0907387

Λb = NestList[(ξb * #) &, 1.0, Nb - 1];

Zone masses (kg)
mb =  $\frac{\Lambda_b m_b[[1]]}{\text{Plus}@@\Lambda_b}$ ;
{mb[[1]], mb[[Nb]]} // ScientificForm
{2.31977 × 10-5, 1.7101 × 10-4}

ρb = 2.5 mp nib

$$\frac{0.018817 \text{ Kilogram}}{\text{Meter}^3}$$


```

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$$r_b = s /. \text{Solve}\left[m_b == \frac{4\pi}{3} (s^3 - r_j^3) \rho_b, s\right][[3]]$$

0.28538 Meter

$$v_{sb} = \text{ionAcousticVelocity}[\mu_b, \gamma_b, 10^{-3} T_{eb}[[1]], z_b] \text{ Meter / Second}$$

$$\frac{43103.8 \text{ Meter}}{\text{Second}}$$

$$v_b = \text{Convert}[100 \text{ Kilo Meter / Second}, \text{Meter / Second}];$$

$$\{\tau_b, \tau_{sb}\} = \frac{\tau_b}{\{v_b, v_{sb}\}}; \text{ScientificForm}[\{\tau_b, \tau_{sb}\}]$$

$$\{(2.8538 \times 10^{-6}) \text{ Second}, (6.62078 \times 10^{-6}) \text{ Second}\}$$

Flux of ions

$$\Gamma_b = n_{ib} v_b // N$$

$$\frac{4.5 \times 10^{29}}{\text{Meter}^2 \text{ Second}}$$

Buffer kinetic energy

$$Q_b = \text{Convert}\left[\frac{m_b}{2} v_b^2, \text{Joule}\right]$$

$$9. \times 10^6 \text{ Joule}$$

■ Summary

□ Lists

$$N_z = \{N_t, N_j, N_b\}$$

$$\{25, 100, 24\}$$

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$$m_{\text{region}} = \{m_t, m_j, m_b\}; \text{Map}[\text{Convert}[\#, \text{Gram}] \&, m_{\text{region}}]$$

$$\{0.000941467 \text{ Gram}, 0.4 \text{ Gram}, 1.8 \text{ Gram}\}$$

$$n_{\text{region}} = \{n_{it}, n_{ij}, n_{ib}\}$$

$$\left\{ \frac{4.3 \times 10^{23}}{\text{Meter}^3}, \frac{8.14944 \times 10^{25}}{\text{Meter}^3}, \frac{4.5 \times 10^{24}}{\text{Meter}^3} \right\}$$

$$Q_{\text{region}} = \{Q_t, Q_j, Q_b\}$$

$$\{0. \text{ Joule}, 3.16907 \times 10^7 \text{ Joule}, 9. \times 10^6 \text{ Joule}\}$$

$$\tau_{\text{region}} = \{\tau_t, \tau_j, \tau_b\}$$

$$\{0. \text{ Second}, 1.85901 \times 10^{-7} \text{ Second}, 2.8538 \times 10^{-6} \text{ Second}\}$$

$$\tau_{s\text{region}} = \{\tau_{st}, \tau_{sj}, \tau_{sb}\}$$

$$\{4.41712 \times 10^{-6} \text{ Second}, 1.71679 \times 10^{-6} \text{ Second}, 6.62078 \times 10^{-6} \text{ Second}\}$$

$$T_{\text{region}} = \{T_{et}, T_{ej}, T_{eb}\}$$

$$\{2 \text{ ElectronVolt}, 29 \text{ ElectronVolt}, 29 \text{ ElectronVolt}\}$$

$$v_{\text{region}} = \{\{0\}, v_j, v_b\}$$

$$\left\{ \{0\}, \frac{398062. \text{ Meter}}{\text{Second}}, \frac{100000 \text{ Meter}}{\text{Second}} \right\}$$

$$\xi_{\text{region}} = \{\xi_t, \xi_j, \xi_b\}$$

$$\{0.05, 0.0613959, 0.0907387\}$$

Total number of zones

$$\text{Plus @@ } N_z$$

$$149$$

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□ Zone widths required for specified number of zones

```

rregion = {rt, rj, rb}
{0.05 Meter, 0.074 Meter, 0.28538 Meter}

Δregion = -Apply[Subtract, Partition[Prepend[rregion, 0], 2, 1], 1]
{0.05 Meter, 0.024 Meter, 0.21138 Meter}

```

□ Summary table

```

lsSummary = {Nz dummyunit, ξregion dummyunit, Δregion, rregion,
  103 mregion, nregion, Tregion, 10-3 vregion, 106 τregion, 106 τsregion, 10-6 Qregion};

TableForm[Map[#[[1]] &, lsSummary, {2}],
  TableHeadings → {{"zones", "zonfac", "Δregion (m)", "r (m)", "m (g)", "n (m-3)",
    "Te (eV)", "v (km/s)", "τ (μs)", "τs (μs)", "KE (MJ)"}, {"Target", "Jets", "Buffer"}}]

```

	Target	Jets	Buffer
zones	25	100	24
zonfac	0.05	0.0613959	0.0907387
Δ _{region} (m)	0.05	0.024	0.21138
r (m)	0.05	0.074	0.28538
m (g)	0.000941467	0.4	1.8
n (m ⁻³)	4.3 × 10 ²³	8.14944 × 10 ²⁵	4.5 × 10 ²⁴
T _e (eV)	2	29	29
v (km/s)	0	398.062	100
τ (μs)	0.	0.185901	2.8538
τ _s (μs)	4.41712	1.71679	6.62078
KE (MJ)	0.	31.6907	9.

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■ Case 030313.02

Based on Thio (1999a). *Using BUCKY routine zoner3 with varying zone masses and thicknesses within a given region. See Subsection "Thio: Current Trends in International Fusion Research 2nd Symposium Paper".*

■ Target

```

μt = 2.5;
Υt = 5 / 3;
nit = 2.0 × 1024 Meter-3;
Nt = 25;
rt = 0.05 Meter;
Tet = 2 ElectronVolt;
zt = 1;
ξt = 0.05;
ξt = 1.0 + ξt
1.05

```

$$m_t = \mu_t m_p n_{it} \frac{4}{3} \pi r_t^3;$$

Total, D, and T masses

$$\{1, 0.4, 0.6\} m_t \frac{10^6 \text{ mg}}{\text{Kilogram}}$$

$$\{4.37892 \text{ mg}, 1.75157 \text{ mg}, 2.62735 \text{ mg}\}$$

```
Λt = NestList[(ξt * #) &, 1.0, Nt - 1];
```

Zone masses (kg)

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```


$$m_t = \frac{\Lambda_t m_t[[1]]}{\text{Plus} @@ \Lambda_t};$$


$$\{m_t[[1]], m_t[N_t]\}$$


$$\{9.17491 \times 10^{-8}, 2.959 \times 10^{-7}\}$$


$$v_{st} = \text{ionAcousticVelocity}[\mu_t, \gamma_t, 10^{-3} T_{et}[[1]], z_t] \text{ Meter / Second}$$


$$\frac{11319.6 \text{ Meter}}{\text{Second}}$$


$$\tau_{st} = \frac{r_t}{v_{st}}$$


$$4.41712 \times 10^{-6} \text{ Second}$$


$$\tau_t = 0.0 \text{ Second};$$


$$Q_t = 0.0 \text{ Joule};$$


```

■ Jet

```


$$\gamma_j = 5 / 3;$$


$$\mu_j = 2.5;$$


$$\eta_j = 60;$$


$$N_j = 100;$$


$$T_{ej} = 26.7 \text{ ElectronVolt};$$


$$z_j = 1;$$


$$m_j = \text{Convert}[0.2 \text{ Gram, Kilogram}]$$


$$0.0002 \text{ Kilogram}$$


$$M_j = m_j / \eta_j$$


$$3.33333 \times 10^{-6} \text{ Kilogram}$$


```

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```


$$\xi_j = x /. \text{FindRoot}\left[\frac{m_j[[1]] / m_t[N_t]}{\text{Plus} @@ \text{NestList}[(x * \#) \&, 1.0, N_j - 1]} == 1, \{x, 1\}\right]$$


$$1.03154$$


$$\xi_j = \xi_j - 1.0$$


$$0.0315417$$


$$\Lambda_j = \text{NestList}[(\xi_j * \#) \&, 1.0, N_j - 1];$$

Zone masses (kg)

$$m_j = \frac{\Lambda_j m_j[[1]]}{\text{Plus} @@ \Lambda_j};$$


$$\{m_j[[1]], m_j[N_j]\} // \text{ScientificForm}$$


$$\{2.959 \times 10^{-7}, 6.40229 \times 10^{-6}\}$$


$$\text{CheckZoneBoundaryMasses}[m_t[N_t], m_j[[1]]]$$


$$r_j = r_t + 0.024 \text{ Meter};$$


$$n_{ij} = v /. \text{Solve}\left[m_j == \mu_j m_p v \frac{4 \pi}{3} (r_j^3 - r_t^3), v\right][[1]]$$


$$\frac{4.07472 \times 10^{25}}{\text{Meter}^3}$$


$$v_{sj} = \text{ionAcousticVelocity}[\mu_j, \gamma_j, 10^{-3} T_{ej}[[1]], z_j] \text{ Meter / Second}$$


$$\frac{41359.2 \text{ Meter}}{\text{Second}}$$

Perfect acoustic match condition:

```

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```

rlul = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
{u1 →  $\frac{125231. \text{ Meter}}{\text{Second}}$ }

vj = u1 /. rlul
 $\frac{125231. \text{ Meter}}{\text{Second}}$ 

{τj, τsj} =  $\frac{r_j}{\{v_j, v_{sj}\}}$ 
{5.90908 × 10-7 Second, 1.7892 × 10-6 Second}

Qj = Convert[ $\frac{m_j}{2} v_j^2$ , Joule]
1.56828 × 106 Joule

Energy per jet
Qj / nj
26138. Joule

```

■ Buffer

```

μb = 2.5;
γb = 5 / 3;
nib = 4.5 × 1024 Meter-3;
Nb = 24;
Teb = Tej;
zb = 1;
mb = Convert[2.0 Gram, Kilogram];

```

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```

ξb = x /. FindRoot[ $\frac{m_b[[1]] / m_j[[N_j]]}{\text{Plus}@@\text{NestList}[(x * \#) \&, 1.0, N_b - 1]} == 1, \{x, 1\}]$ 
1.18501

ξb = ξb - 1.0
0.185012

Λb = NestList[(ξb * #) &, 1.0, Nb - 1];

Zone masses (kg)
mb =  $\frac{\Lambda_b m_b[[1]]}{\text{Plus}@@\Lambda_b}$ ;
{mb[[1]], mb[[Nb]]} // ScientificForm
{6.40229 × 10-6, 3.17656 × 10-4}

CheckZoneBoundaryMasses[mj[[Nj]], mb[[1]]]

ρb = μb mp nib
 $\frac{0.018817 \text{ Kilogram}}{\text{Meter}^3}$ 

rb = s /. Solve[mb ==  $\frac{4 \pi}{3} (s^3 - r_j^3) \rho_b, s]$ [[3]]
0.295409 Meter

vsb = ionAcousticVelocity[μb, γb, 10-3 Teb[[1]], zb] Meter / Second
 $\frac{41359.2 \text{ Meter}}{\text{Second}}$ 

```

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```

vb = vj
125231. Meter
Second

{τb, τsb} =  $\frac{\mathbf{r_b}}{\{\mathbf{v_b}, \mathbf{v_{sb}}\}}$ ; ScientificForm[{τb, τsb}]
{(2.35891 × 10-6) Second, (7.14253 × 10-6) Second}

```

Flux of ions

```

Γb = nib vb // N
5.6354 × 1029
Meter2 Second

```

Buffer kinetic energy

```

Qb = Convert[ $\frac{\mathbf{m_b}}{2} \mathbf{v_b^2}$ , Joule]
1.56828 × 107 Joule

```

■ Summary

□ Setup print lists

```

Nz = {Nt, Nj, Nb};
mregion = {mt, mj, mb};
nregion = {nit, nij, nib};
Qregion = {Qt, Qj, Qb};
τregion = {τt, τj, τb};
τsregion = {τst, τsj, τsb};

```

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```

Tregion = {Tet, Tej, Teb};
vregion = {{0}, vj, vb};
ξregion = {ξt, ξj, ξb};

Total number of zones

Plus @@ Nz
149

```

□ Zone widths required for specified number of zones

```

rregion = {rt, rj, rb};
Δregion = -Apply[Subtract, Partition[Prepend[rregion, 0.0 Meter], 2, 1], 1];

```

□ Summary table

```

lsSummary = {Nz dummyunit, ξregion dummyunit, Δregion, rregion,
103 mregion, nregion, Tregion, 10-3 vregion, 106 τregion, 106 τsregion, 10-6 Qregion};

```

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```
TableForm[Map[#[[1]] &, lsSummary, {2}],
  TableHeadings → {{ "zones", "zonfac", "Δregion(m)", "r(m)", "m(g)", "n(m-3)",
    "Te(eV)", "v(km/s)", "τ(μs)", "τs(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]

```

	Target	Jets	Buffer
zones	25	100	24
zonfac	0.05	0.0315417	0.185012
Δ _{region} (m)	0.05	0.024	0.221409
r(m)	0.05	0.074	0.295409
m(g)	0.00437892	0.2	2.
n(m ⁻³)	2. × 10 ²⁴	4.07472 × 10 ²⁵	4.5 × 10 ²⁴
T _e (eV)	2	26.7	26.7
v(km/s)	0	125.231	125.231
τ(μs)	0.	0.590908	2.35891
τ _s (μs)	4.41712	1.7892	7.14253
KE(MJ)	0.	1.56828	15.6828

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■ Case 030313.03

Case 030313.02 with v_{jet}=250 km/s, maintaining "perfect acoustic matching."

■ Target

```
μt = 2.5;
γt = 5 / 3;
nit = 2.0 × 1024 Meter-3;
Nt = 25;
rt = 0.05 Meter;
Tet = 2 ElectronVolt;
zt = 1;
ξt = 0.05;
ξt = 1.0 + ξt
1.05
```

$$m_t = \mu_t m_p n_{it} \frac{4}{3} \pi r_t^3;$$

Total, D, and T masses

$$\{1, 0.4, 0.6\} m_t \frac{10^6 \text{ mg}}{\text{Kilogram}}$$

$$\{4.37892 \text{ mg}, 1.75157 \text{ mg}, 2.62735 \text{ mg}\}$$

$$\Lambda_t = \text{NestList}[(\xi_t * \#) \&, 1.0, N_t - 1];$$

Zone masses (kg)

$$m_t = \frac{\Lambda_t m_t[[1]]}{\text{Plus @@ } \Lambda_t};$$

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```

{m_t[[1]], m_t[[N_t]]}
{9.17491 × 10-8, 2.959 × 10-7}

v_st = ionAcousticVelocity[μ_t, γ_t, 10-3 T_et[[1]], z_t] Meter / Second
11319.6 Meter
Second

τ_st =  $\frac{r_t}{v_{st}}$ 
4.41712 × 10-6 Second

τ_t = 0.0 Second;
Q_t = 0.0 Joule;

```

■ Jet

```

γ_j = 5 / 3;
μ_j = 2.5;
η_j = 60;
N_j = 100;
T_ej = 106 ElectronVolt;
z_j = 1;

m_j = Convert[0.2 Gram, Kilogram]
0.0002 Kilogram

M_j = m_j / η_j
3.33333 × 10-6 Kilogram

```

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```

ξ_j = x /. FindRoot[ $\frac{m_j[[1]] / m_t[[N_t]]}{\text{Plus} @@ \text{NestList}[(x * \#) \&, 1.0, N_j - 1]} == 1, \{x, 1\}]$ 
1.03154

ξ_j = ξ_j - 1.0
0.0315417

Λ_j = NestList[(ξ_j * #) &, 1.0, N_j - 1];
Zone masses (kg)
m_j =  $\frac{\Lambda_j m_j[[1]]}{\text{Plus} @@ \Lambda_j}$ ;
{m_j[[1]], m_j[[N_j]]} // ScientificForm
{2.959 × 10-7, 6.40229 × 10-6}

CheckZoneBoundaryMasses[m_t[[N_t]], m_j[[1]]]

r_j = r_t + 0.024 Meter;

n_ij = v /. Solve[m_j == μ_j m_p v  $\frac{4 \pi}{3} (r_j^3 - r_t^3)$ , v][[1]]
 $\frac{4.07472 \times 10^{25}}{\text{Meter}^3}$ 

v_sj = ionAcousticVelocity[μ_j, γ_j, 10-3 T_ej[[1]], z_j] Meter / Second
82407.9 Meter
Second

Perfect acoustic match condition:

```

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```

rlul = Solve[equal /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
{u1 →  $\frac{249522 \text{ Meter}}{\text{Second}}$ }

vj = u1 /. rlul
 $\frac{249522 \text{ Meter}}{\text{Second}}$ 

{τj, τsj} =  $\frac{r_j}{\{v_j, v_{sj}\}}$ 
{2.96567 × 10-7 Second, 8.97972 × 10-7 Second}

Qj = Convert[ $\frac{m_j}{2} v_j^2$ , Joule]
6.22613 × 106 Joule

Energy per jet
Qj / nj
103769. Joule

```

■ Buffer

```

μb = 2.5;
γb = 5 / 3;
nib = 4.5 × 1024 Meter-3;
Nb = 24;
Teb = Tej;
zb = 1;
mb = Convert[2.0 Gram, Kilogram];

```

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```

ξb = x /. FindRoot[ $\frac{m_b[[1]] / m_j[[N_j]]}{\text{Plus}@@\text{NestList}[(x * \#) \&, 1.0, N_b - 1]} == 1, \{x, 1\}]$ 
1.18501

ξb = ξb - 1.0
0.185012

Λb = NestList[(ξb * #) &, 1.0, Nb - 1];

Zone masses (kg)
mb =  $\frac{\Lambda_b m_b[[1]]}{\text{Plus}@@\Lambda_b}$ ;
{mb[[1]], mb[[Nb]]} // ScientificForm
{6.40229 × 10-6, 3.17656 × 10-4}

CheckZoneBoundaryMasses[mj[[Nj]], mb[[1]]]

ρb = μb mp nib
 $\frac{0.018817 \text{ Kilogram}}{\text{Meter}^3}$ 

rb = s /. Solve[mb ==  $\frac{4 \pi}{3} (s^3 - r_j^3) \rho_b$ , s] [[3]]
0.295409 Meter

vsb = ionAcousticVelocity[μb, γb, 10-3 Teb[[1]], zb] Meter / Second
 $\frac{82407.9 \text{ Meter}}{\text{Second}}$ 

```

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```

vb = vj

$$\frac{249522. \text{ Meter}}{\text{Second}}$$

{ $\tau_b$ ,  $\tau_{sb}$ } =  $\frac{\mathbf{r_b}}{\{\mathbf{v_b}, \mathbf{v_{sb}}\}}$ ; ScientificForm[{ $\tau_b$ ,  $\tau_{sb}$ }]
{ (1.1839 × 10-6) Second, (3.58472 × 10-6) Second}

```

Flux of ions

```

Γb = nib vb // N

$$\frac{1.12285 \times 10^{30}}{\text{Meter}^2 \text{ Second}}$$


```

Buffer kinetic energy

```

Qb = Convert[ $\frac{\mathbf{m_b}}{2} \mathbf{v_b^2}$ , Joule]
6.22613 × 107 Joule

```

■ Summary

□ Setup print lists

```

Nz = {Nt, Nj, Nb};
mregion = {mt, mj, mb};
nregion = {nit, nij, nib};
Qregion = {Qt, Qj, Qb};
τregion = {τt, τj, τb};
τsregion = {τst, τsj, τsb};

```

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```

Tregion = {Tet, Tej, Teb};
vregion = {{0}, vj, vb};
ξregion = {ξt, ξj, ξb};
Total number of zones
Plus @@ Nz
149

```

□ Zone widths required for specified number of zones

```

rregion = {rt, rj, rb};
Δregion = -Apply[Subtract, Partition[Prepend[rregion, 0.0 Meter], 2, 1], 1];

```

□ Summary table

```

lsSummary = {Nz dummyunit, ξregion dummyunit, Δregion, rregion,
  103 mregion, nregion, Tregion, 10-3 vregion, 106 τregion, 106 τsregion, 10-6 Qregion};

```

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```
TableForm[Map[#[[1]] &, lsSummary, {2}],
  TableHeadings -> {{ "zones", "zonfac", "Δregion(m)", "r(m)", "m(g)", "n(m-3)",
    "Te(eV)", "v(km/s)", "τ(μs)", "τs(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]

```

	Target	Jets	Buffer
zones	25	100	24
zonfac	0.05	0.0315417	0.185012
Δ _{region} (m)	0.05	0.024	0.221409
r(m)	0.05	0.074	0.295409
m(g)	0.00437892	0.2	2.
n(m ⁻³)	$2. \times 10^{24}$	4.07472×10^{25}	4.5×10^{24}
T _e (eV)	2	106	106
v(km/s)	0	249.522	249.522
τ(μs)	0.	0.296567	1.1839
τ _s (μs)	4.41712	0.897972	3.58472
KE(MJ)	0.	6.22613	62.2613

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BUCKY cases 030121.06&7 (D-T, perfect acoustic matching, $v_{jet}=400$ km/s)

★ *Note: the compression for this case is too good to be true, because the masses were not matched well at the interfaces, but this is the sort of behavior that is sought. The case is left here as a potential starting point for other calculations.*

★ *Note: the full set of commands to generate the plots is shown in the section "BUCKY case 03013.02" below. In this section only the plots and the outline of the structure appear.*

■ Setup

```
caseID = "030121_06";

params = Graphics[Text["Case " <> caseID <> "\n
mt=0.1 mg; Δt=3.1 cm\n
mj=0.4 g; Δj=3.6 cm\n
mb=1.8 g; Δb=21 cm\n
vj=398 km/s; vb=100 km/s", {0, 0}, {0, 0}]]];

```

■ 0-1 μs

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\0_1_us\\";

```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 1000;

```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";

thickness = Thickness[0.0002];

```

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- ☐ Read data
- ☐ Number of time steps
- ☐ Number and list of radial zones in file
- ☐ Convert data from {s, cm} to {μs, m}
- ☐ Plots
- $r < 0.1$ m

```

SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.1}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.024}, {0.95 * 10-3 tEnd, 0.028}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}];

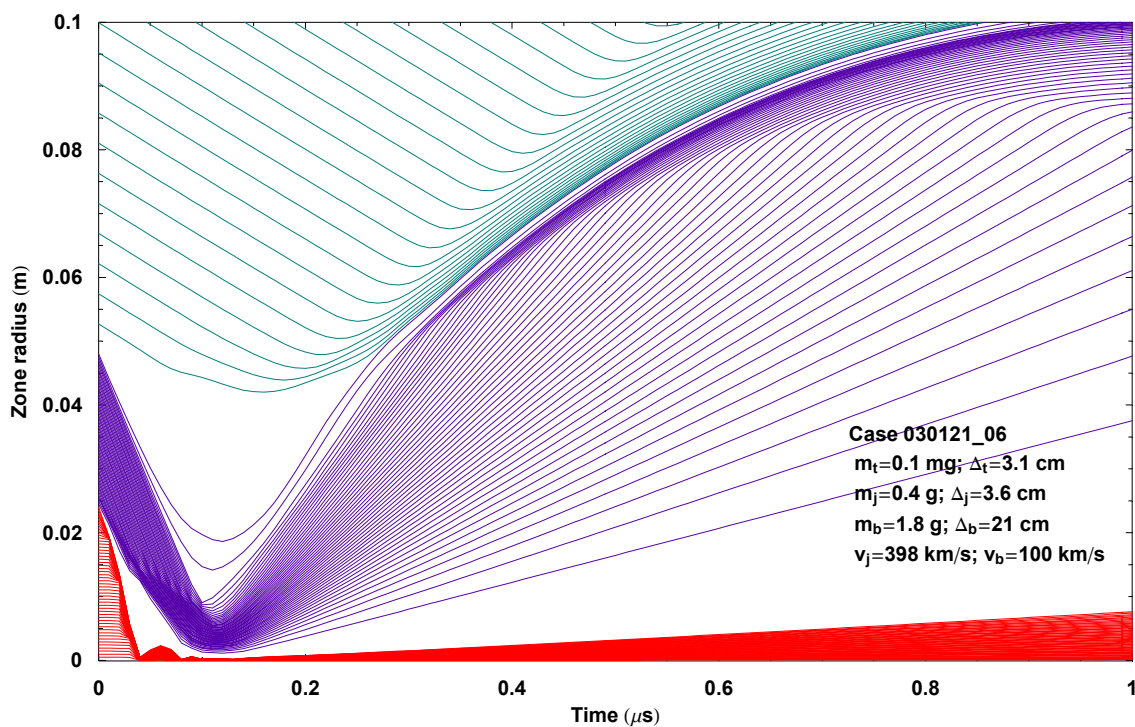
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



- ☐ Undo options

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■ Neutron yield vs time (PLOT.NeutronsYield.vs.t)

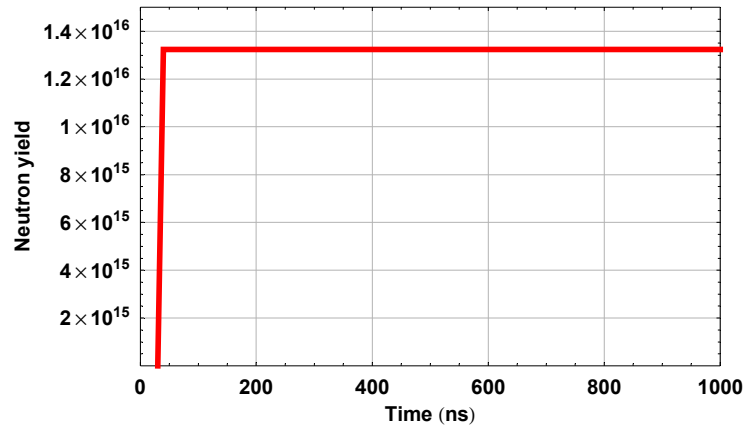
□ Set up problem and import data

□ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {0.999 × 1012, 1.5 × 1016}};
p11s = ListPlot[lsData[plotID], plotrange, PlotJoined → True];

```



□ Undo options

■ Density vs time in each region (PLOT.regn.dens)

□ Set up problem and import data

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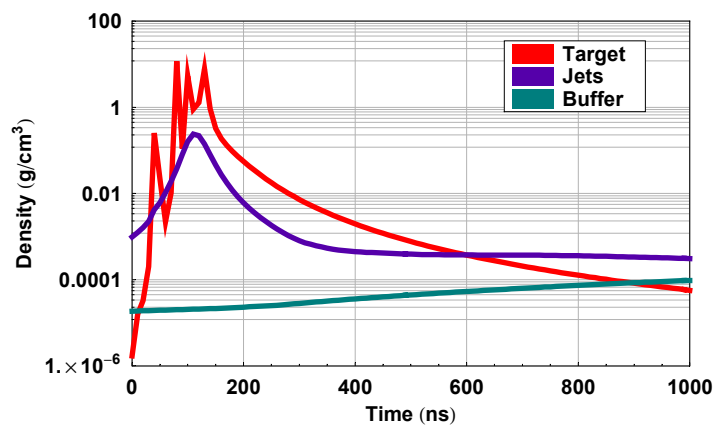
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□ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {10-6.001, 1 × 102.001}};
p11s = Table[LogListPlot[lsData[plotID][[All, i]], plotrange, PlotJoined → True,
  PlotStyle → lineStyle[[i, All]], DisplayFunction → Identity], {i, 1, 3}];
ShowLegend[Show[p11s],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.4, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



□ Undo options

JFS

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■ Ion temperature vs time in each region (PLOT.regn.temp)

□ Set up problem and import data

□ Plot

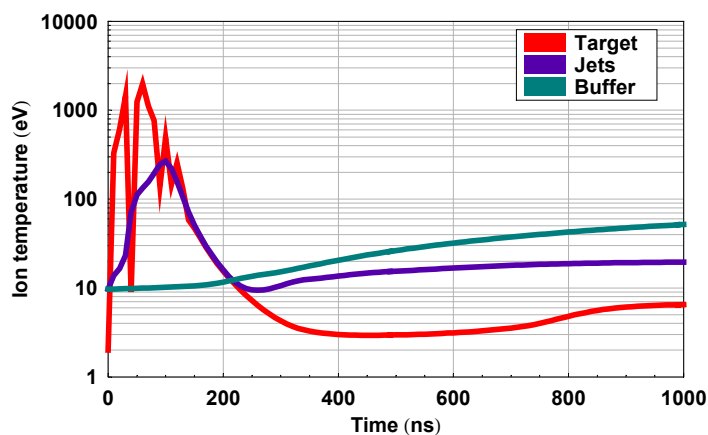
```

plotrange = PlotRange → {{-10-3, tEnd}, {0.999, 104}};

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[p11s],
  {{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}]},
  LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



JFS

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17:03:59

□ Undo options

■ Kinetic energy vs time in each region (PLOT.regn.KEs)

□ Set up problem and import data

□ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {10-5.0001, 100}};

gridlinesY = Map[#, {GrayLevel[0.7]}] &, Flatten[Table[Log[10, j * 10i], {i, -5, 2}, {j, 1, 9}]]];

frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

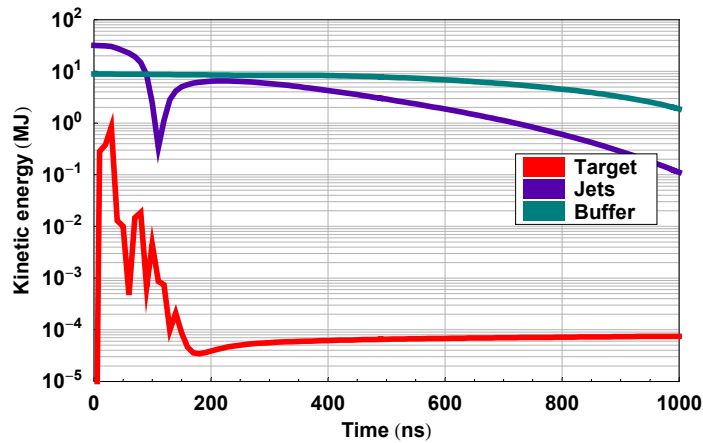
```

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```
ShowLegend[Show[p1ls, GridLines -> {{Automatic, GrayLevel[0.7]}, gridlinesY},
  FrameTicks -> {Automatic, frameticksY, None, None}],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition -> {0.45, 0}, LegendSize -> {0.4, 0.2}, LegendShadow -> {0, 0}}];
```



☐ Undo options

☒ Ion temperature vs radius (PLOT.tempn)

☒ Mass density vs radius (PLOT.rho)

☒ Fluid velocity vs radius (PLOT.vel)

☒ Fusion power vs radius (BPLOT variables 30 vs 1)

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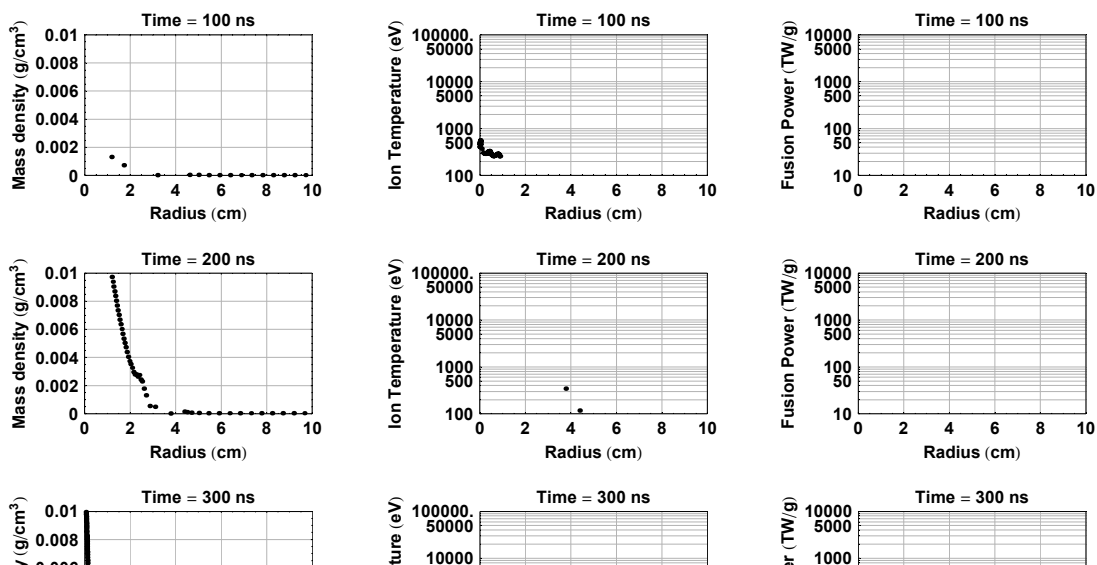
☒ Combined plots: mass density, ion temperature, and fusion power vs radius

☐ Setup

☐ Table of plots

☐ Array of plots

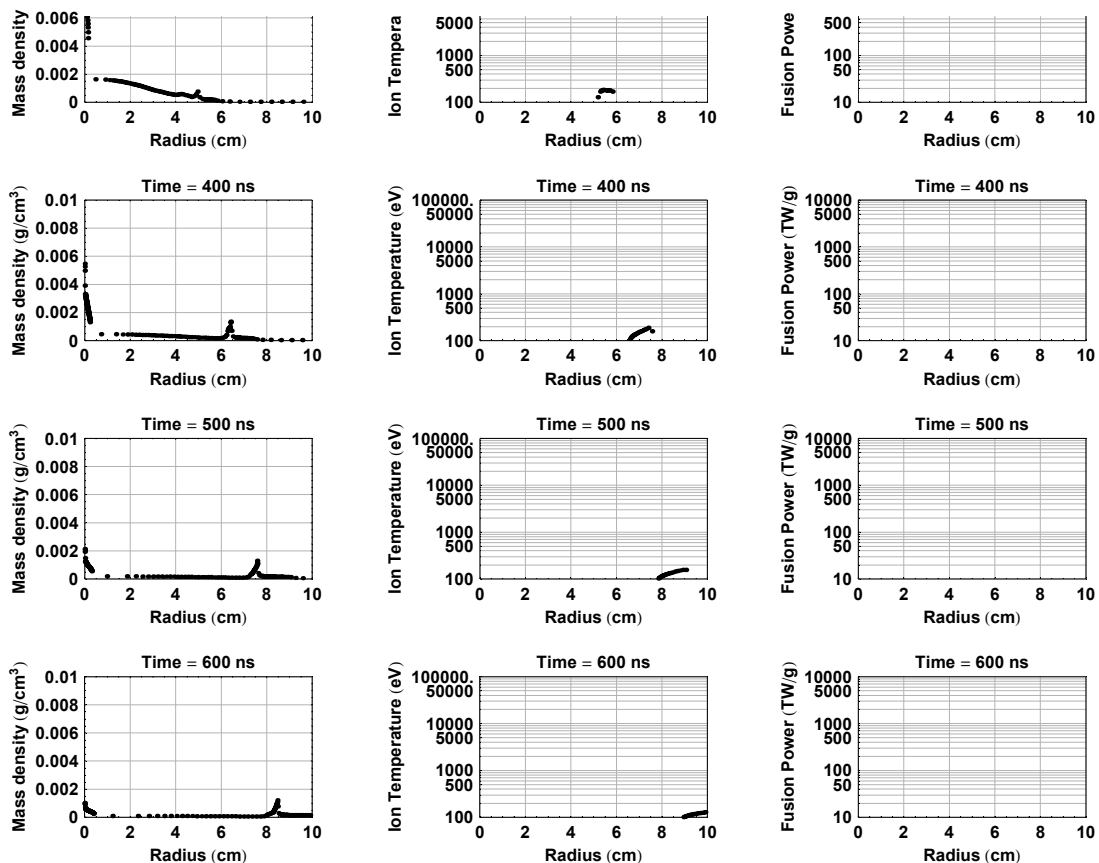
```
Show[GraphicsArray[Transpose[{tb1, tb2, tb3}]], DisplayFunction -> $DisplayFunction];
```



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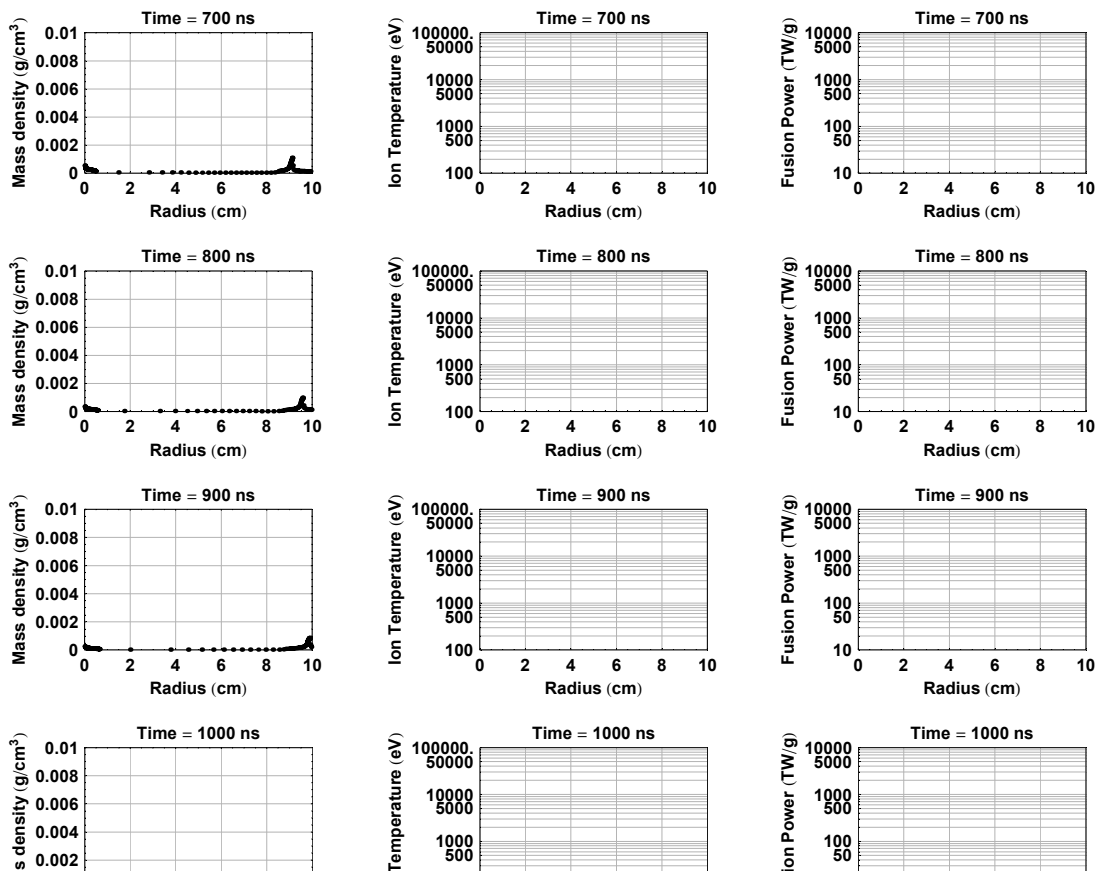
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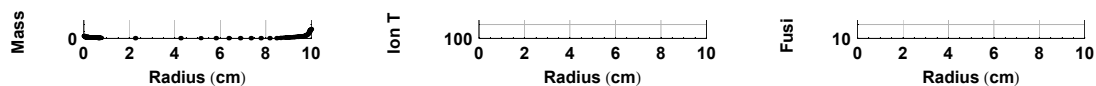
17:03:59



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17:04:00



☐ Undo options

JFS

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■ 0-100 ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\0_100_ns\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 100;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

☐ Plots

● r < 0.03 m

```
SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.03}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.024}, {0.95 * 10-3 tEnd, 0.028}, params],
  DisplayFunction → Identity];

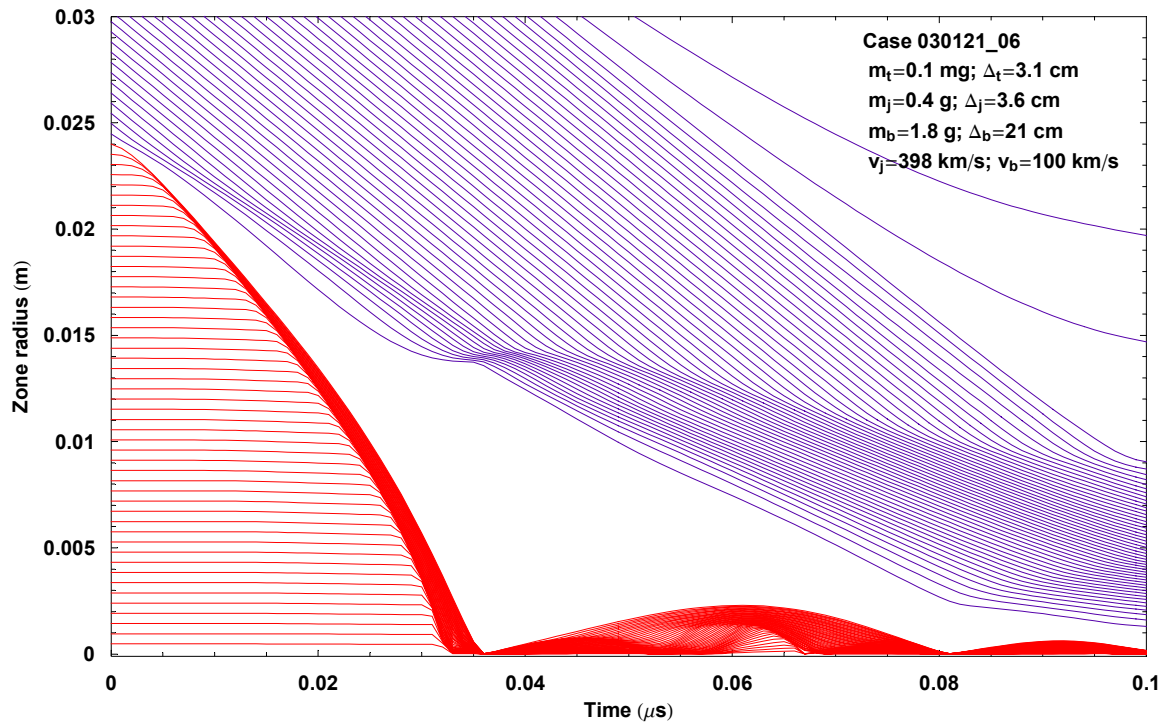
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}];
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction -> $DisplayFunction];
```



JFS

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• $r < 0.005$ m

```
SetOptions[ListPlot, PlotRange -> {{-10-6, 10-3 tEnd}, {-10-4, 0.005}},
  PlotJoined -> True, GridLines -> None, FrameLabel -> {"Time (μs)", "Zone radius (m)"},
  Epilog -> Rectangle[{0.4 * 10-3 tEnd, 0.003}, {0.6 * 10-3 tEnd, 0.004}, params],
  DisplayFunction -> Identity];

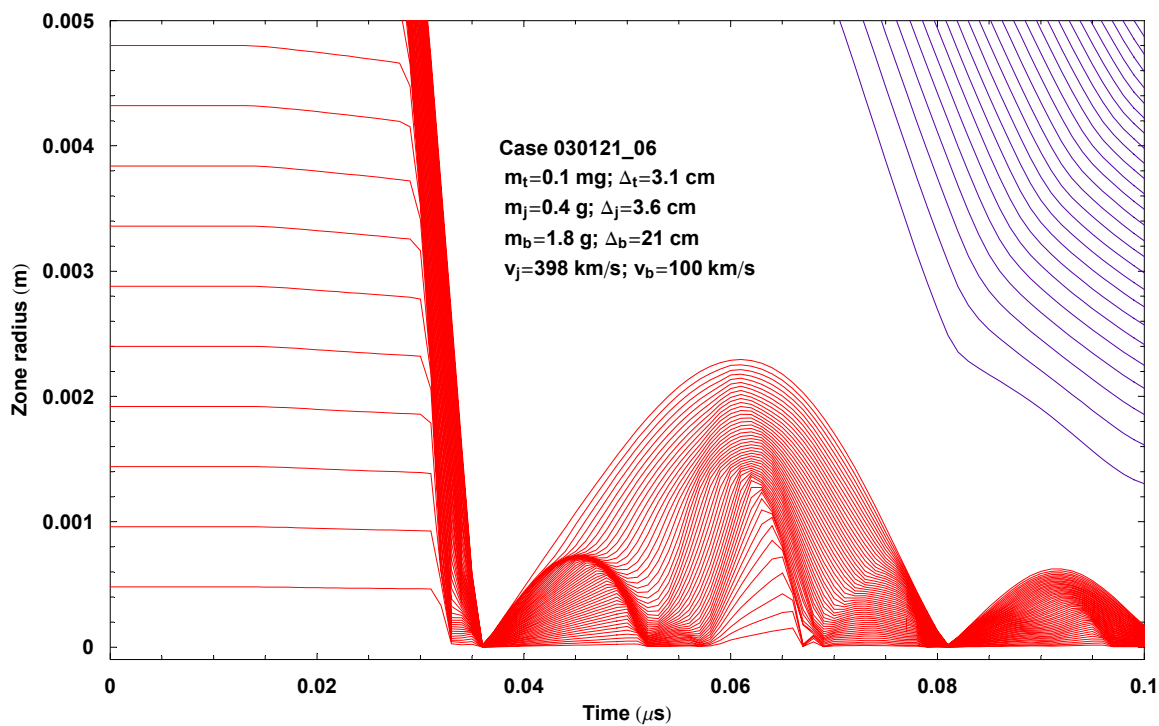
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle -> red], {i, 1, N_t}];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle -> purple], {i, N_t + 1, N_t + N_j}];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle -> teal], {i, N_t + N_j + 1, N_z}];
```

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Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → \$DisplayFunction];



☐ Undo options

JFS

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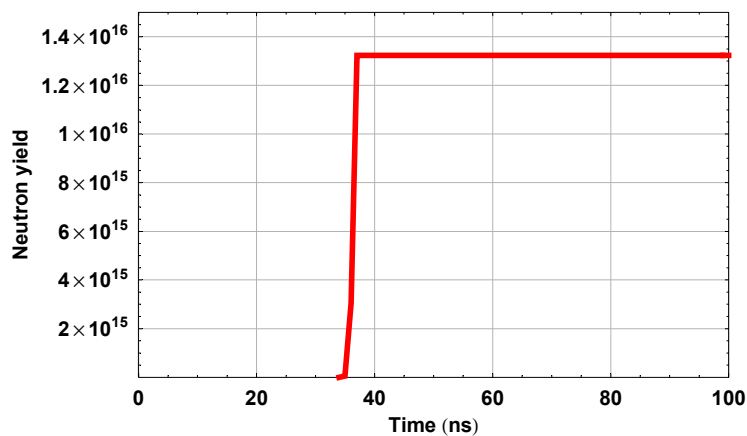
17:04:00

■ Neutron yield vs time (PLOT.NeutronsYield.vs.t)

☐ Set up problem and import data

☐ Plot

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → {lineStyle[1, All]}, FrameLabel → {"Time (ns)", "Neutron yield"}];
plotrange = PlotRange → {{-10-3, tEnd}, {0.999 × 1012, 1.5 × 1016}};
p1ls = ListPlot[lsData[plotID], plotrange, PlotJoined → True];
```



☐ Undo options

JFS

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17:04:00

■ Density vs time in each region (PLOT.regn.dens)

□ Set up problem and import data

□ Plot

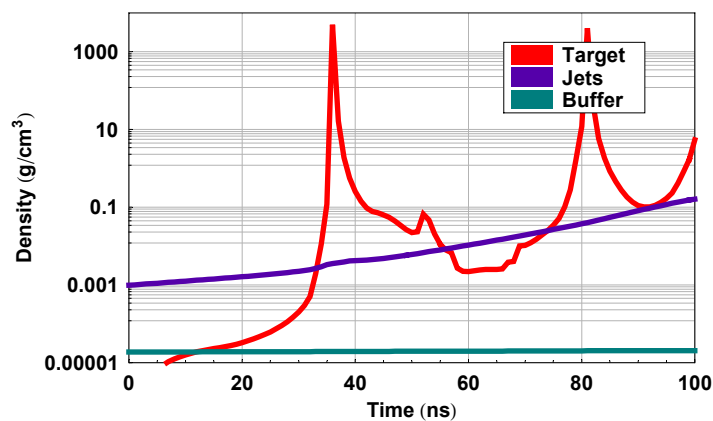
```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm3)"}];
plotrange = PlotRange → {{-10-3, tEnd}, {10-5.001, 1 × 103.999}};
p1ls = Table[LogListPlot[lsData[plotID][[All, i]], plotrange, PlotJoined → True,
  PlotStyle → lineStyle[[i, All]], DisplayFunction → Identity], {i, 1, 3}];
```

JFS

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```
ShowLegend[Show[p1ls],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.4, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



□ Undo options

■ Ion temperature vs time in each region (PLOT.regn.temp)

□ Set up problem and import data

□ Plot

```
plotrange = PlotRange → {{-10-3, tEnd}, {0.999, 105}};
```

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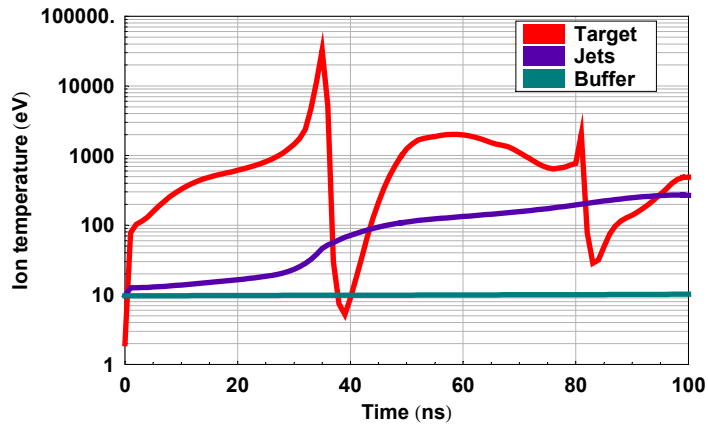
17:04:00

```

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[p11s],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



☐ Undo options

☒ Kinetic energy vs time in each region (PLOT.regn.KEs)

☐ Set up problem and import data

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☐ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {10-5.0001, 100}};

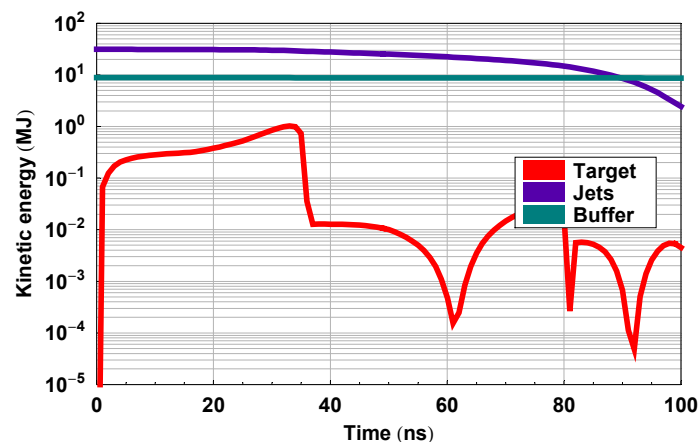
gridlinesY = Map[#, {GrayLevel[0.7]}] &, Flatten[Table[Log[10, j * 10i], {i, -5, 2}, {j, 1, 9}]]];

frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[p11s, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
  FrameTicks → {Automatic, frameticksY, None, None}],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



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□ Undo options

■ Ion temperature vs radius (PLOT.tempn)

■ Mass density vs radius (PLOT.rho)

■ Fluid velocity vs radius (PLOT.vel)

■ Fusion power vs radius (BPlot variables 30 vs 1)

■ Combined plots: mass density, ion temperature, and fusion power vs radius

□ Setup

□ Table of plots

Partition length

```
partitionL = 3;
```

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → PointSize[0.02], DefaultFont → {"Helvetica-Bold", 10}];
```

```
tb1 = Table[ListPlot[lsData[plotID1][[i]],
  plotrangeRho, PlotLabel → "Time = "<>ToString[Round[lsTimes[[i]]]<>" ns",
  FrameLabel → {"Radius (cm)", "Mass density (g/cm³)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID1]]}];
```

```
tb2 = Table[LogListPlot[lsData[plotID2][[i]],
  plotrangeTi, PlotLabel → "Time = "<>ToString[Round[lsTimes[[i]]]<>" ns",
  FrameLabel → {"Radius (cm)", "Ion Temperature (eV)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID2]]}];
```

JFS

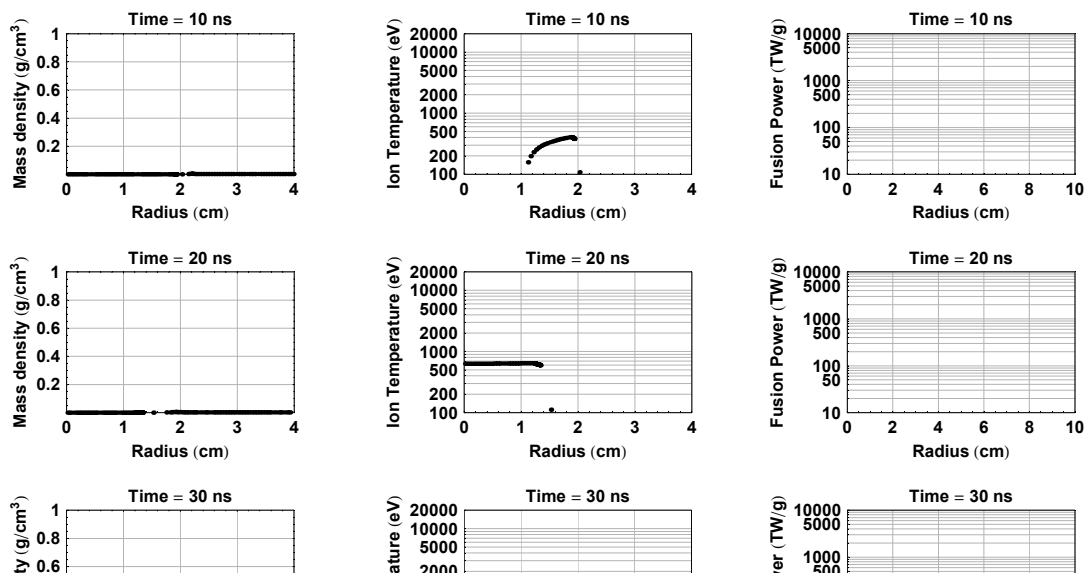
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```
tb3 = Table[LogListPlot[lsData[plotID3][[i]],
  plotrangePfus, PlotLabel → "Time = "<>ToString[Round[lsTimes[[i]]]<>" ns",
  FrameLabel → {"Radius (cm)", "Fusion Power (TW/g)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID3]]}];
```

□ Array of plots

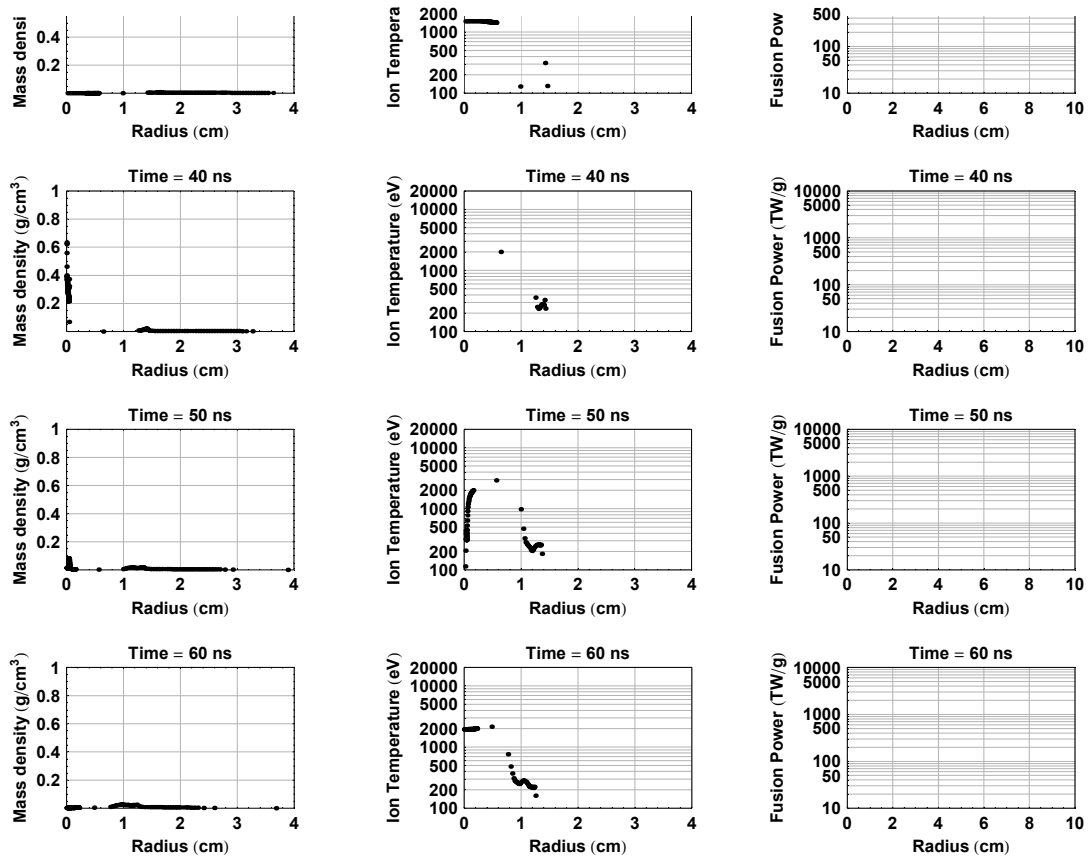
```
Show[GraphicsArray[Transpose[{tb1, tb2, tb3}]], DisplayFunction → $DisplayFunction];
```



JFS

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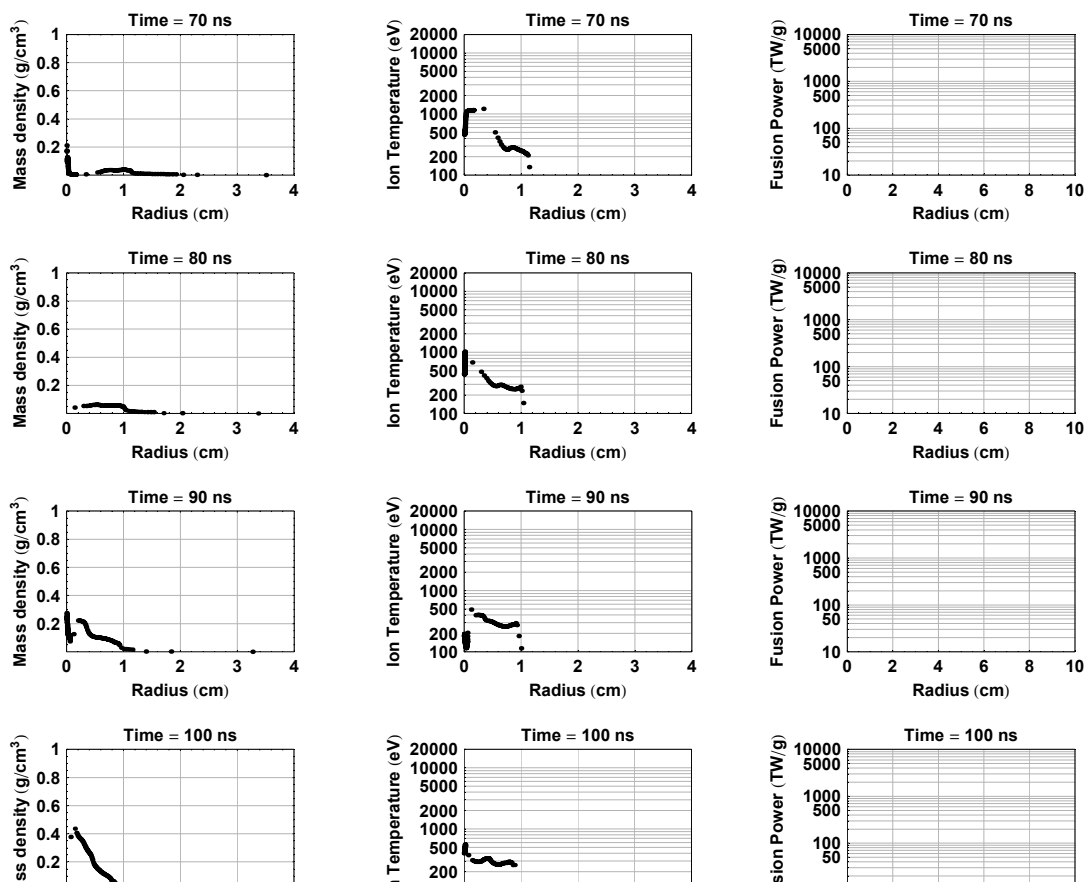
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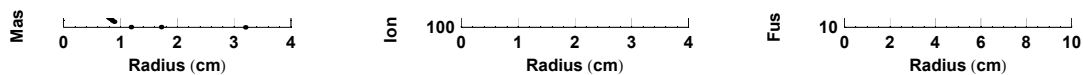
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☐ Undo options

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BUCKY case 030201.05 (D-T, perfect acoustic matching, 149 zones, $v_{jet}=398$ km/s)

★ Note: the full set of commands to generate the plots is shown in the section "BUCKY case 030313.02" below. In this section only the plots and the outline of the structure appear.

■ Setup

```
caseID = "030201_05";

params = Graphics[Text["Case " <> caseID <> "\n
mt=1.0 mg; Δt=5.0 cm\n
mj=0.4 g; Δj=2.4 cm\n
mb=1.8 g; Δb=21.1 cm\n
vj=398 km/s; vb=100 km/s", {0, 0}, {0, 0}]];
```

■ 0-200ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 200;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";

thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

JFS

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□ Number and list of radial zones in file

□ Convert data from {s, cm} to {μs, m}

□ Plots

• $r < 0.3$ m

```
SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
  Rectangle[{0.75 * 10-3 tEnd, 0.07}, {0.95 * 10-3 tEnd, 0.17}, params], DisplayFunction → Identity];

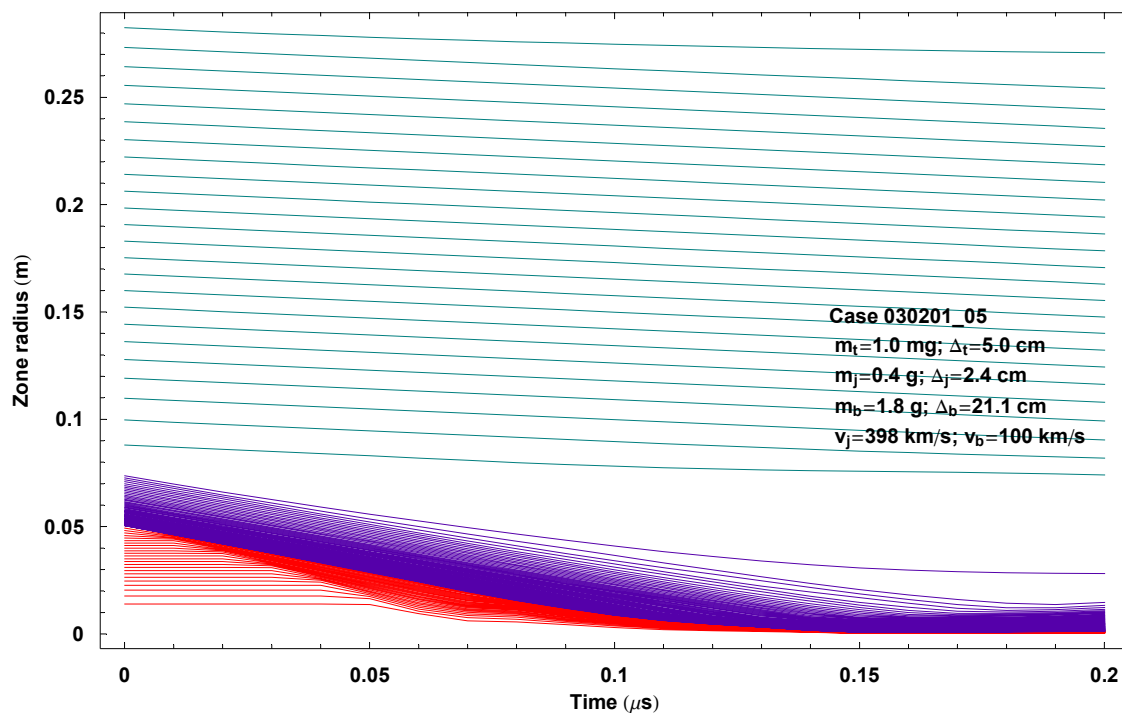
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}];
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



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• $r < 0.05$ m

```
SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.035}, {0.95 * 10-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];

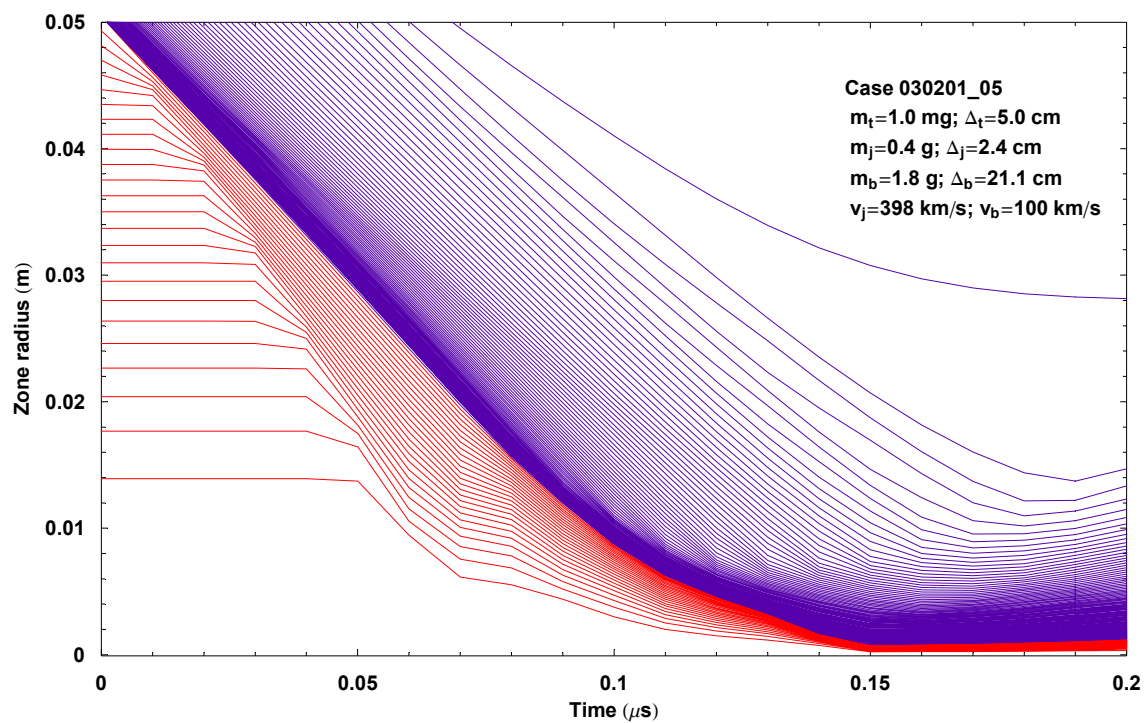
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}};
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}};
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}};
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



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• $r < 0.02$ m

```

SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.02}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.7 * 10-3 tEnd, 0.014}, {0.9 * 10-3 tEnd, 0.018}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt]];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj]];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz]];

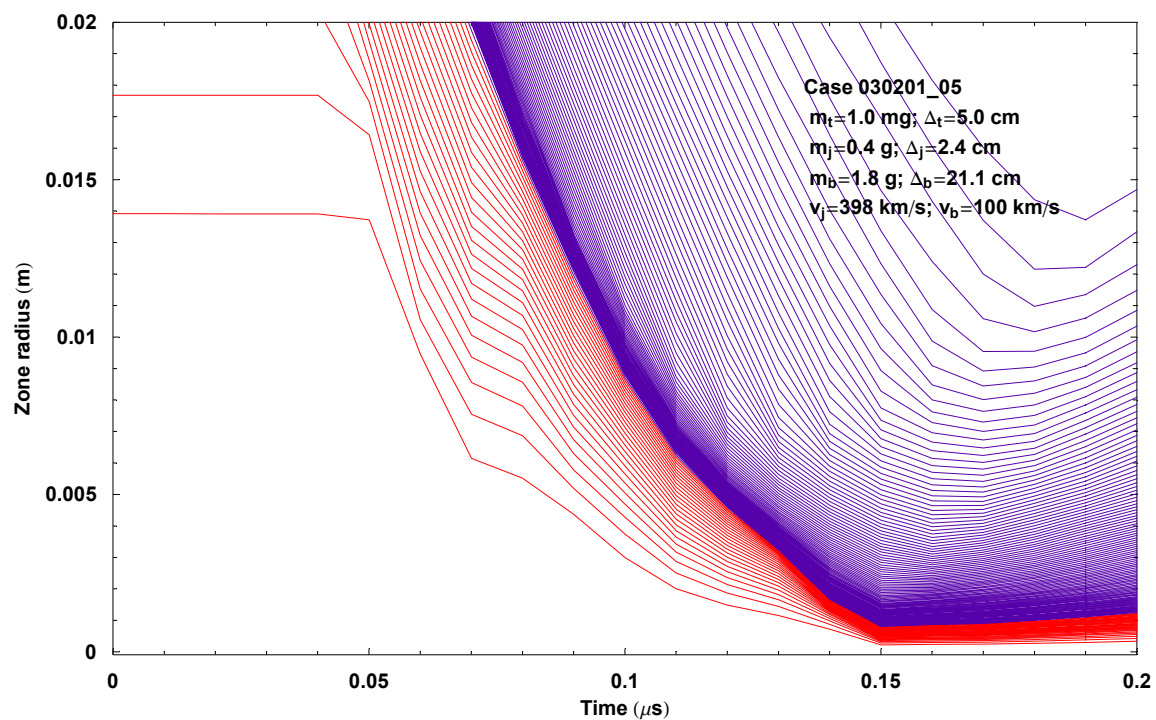
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



☐ Undo options

JFS

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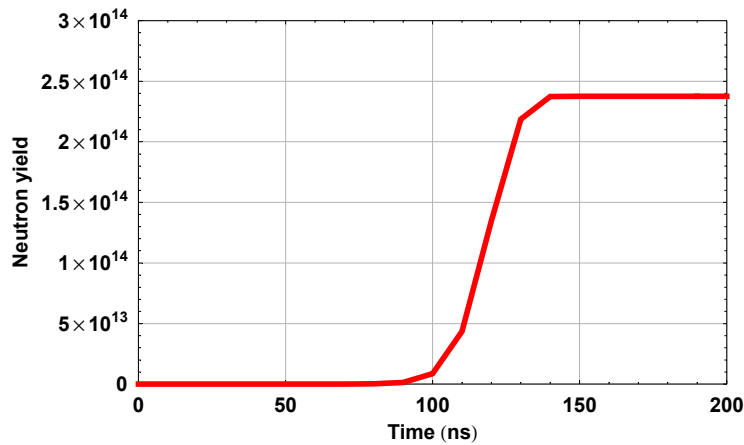
17:04:01

■ Neutron yield vs time (PLOT.NeutronsYield.vs.t)

□ Set up problem and import data

□ Plot

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → lineStyle[1, All], FrameLabel → {"Time (ns)", "Neutron yield"}];
plotrange = PlotRange → {{-10-3, tEnd}, {-1011, 3 × 1014}};
p11s = ListPlot[lsData[plotID], plotrange, PlotJoined → True];
```



□ Undo options

JFS

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17:04:01

■ Density vs time in each region (PLOT.regn.dens)

□ Set up problem and import data

□ Plot

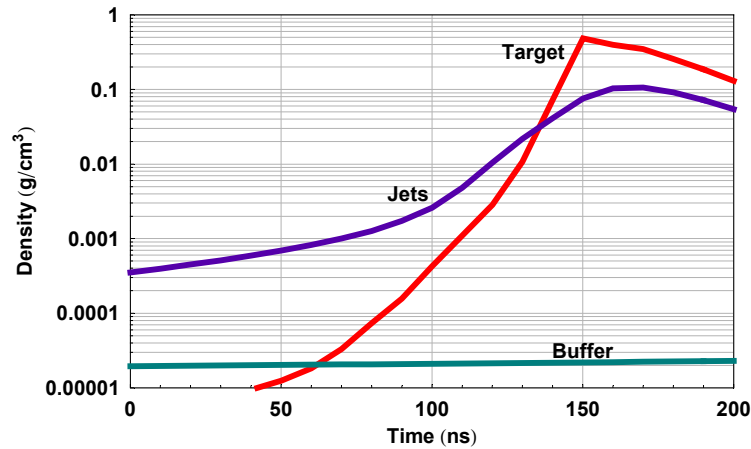
```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm3)"}];
plotrange = PlotRange → {{-10-3, tEnd}, {10-5.001, 1}};
p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange, PlotJoined → True,
  PlotStyle → lineStyle[i, All], DisplayFunction → Identity], {i, 1, 3}];
text = {Graphics[Text["Target", {145, -0.5}, {1, 0}]],
  Graphics[Text["Jets", {100, -2.4}, {1, 0}]],
  Graphics[Text["Buffer", {150, -4.5}, {0, 0}]]};
```

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```
Show[p11s, text,
  FrameLabel → {"Time (ns)", "Density (g/cm³)"}, DisplayFunction → $DisplayFunction];
```



☐ Undo options

☒ Ion temperature vs time in each region (PLOT.regn.temp)

☐ Set up problem and import data

☐ Plot

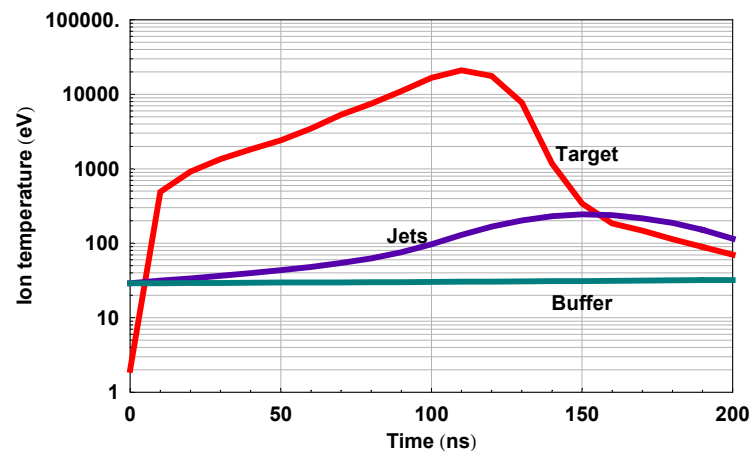
```
plotrange = PlotRange → {{-10-3, tEnd}, {0.999, 105}};
```

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```
p11s = Table[LogListPlot[lsData[plotID][[All, i]], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];
text = {Graphics[Text["Target", {140, 3.2}, {-1, 0}]],
  Graphics[Text["Jets", {100, 2.1}, {1, 0}]],
  Graphics[Text["Buffer", {150, 1.2}, {0, 0}]]};
Show[p11s, text, FrameLabel → {"Time (ns)", "Ion temperature (eV)"},
  DisplayFunction → $DisplayFunction];
```



☐ Undo options

JFS

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17:04:01

■ Kinetic energy vs time in each region (PLOT.regn.KEs)

□ Set up problem and import data

□ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {10-5.0001, 100}};
gridlinesY = Map[#, {GrayLevel[0.7]}] &, Flatten[Table[Log[10, j * 10i], {i, -5, 2}, {j, 1, 9}]]];
frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];
p11s = Table[LogListPlot[lsData[plotID][[All, i]], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

```

JFS

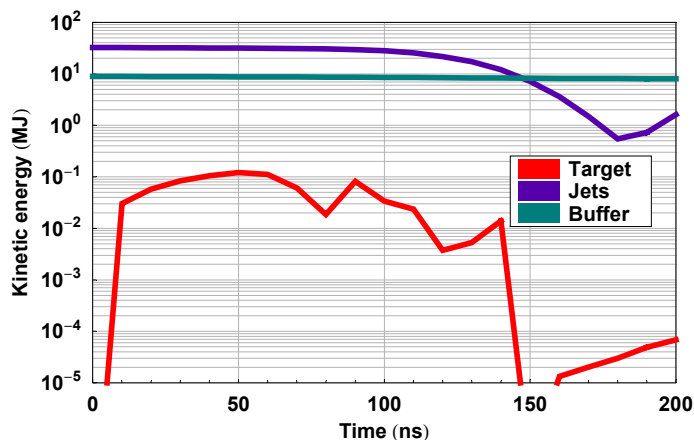
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```

ShowLegend[Show[p11s, GridLines → {{Automatic, GrayLevel[0.7]}}, gridlinesY],
  FrameTicks → {Automatic, frameticksY, None, None}],
{{lineStyle[[1, 1]], "Target"}, {lineStyle[[2, 1]], "Jets"}, {lineStyle[[3, 1]], "Buffer"}},
  LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}];

```



□ Undo options

■ Ion temperature vs radius (PLOT.tempn)

■ Mass density vs radius (PLOT.rho)

■ Fluid velocity vs radius (PLOT.vel)

■ Fusion power vs radius (BPLOT variables 30 vs 1)

JFS

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■ Combined plots: mass density, ion temperature, and fusion power vs radius

□ Setup

□ Table of plots

Partition length

```
partitionL = 3;
```

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → PointSize[0.02], DefaultFont → {"Helvetica-Bold", 10}];

tb1 = Table[ListPlot[lsData[plotID1][[i]],
  plotrangeRho, PlotLabel → "Time = " <> ToString[Round[lsTimes[[i]]] <> " ns",
  FrameLabel → {"Radius (cm)", "Mass density (g/cm³)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID1]}}];

tb2 = Table[LogListPlot[lsData[plotID2][[i]],
  plotrangeTi, PlotLabel → "Time = " <> ToString[Round[lsTimes[[i]]] <> " ns",
  FrameLabel → {"Radius (cm)", "Ion Temperature (eV)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID2]}}];

tb3 = Table[LogListPlot[lsData[plotID3][[i]],
  plotrangePfus, PlotLabel → "Time = " <> ToString[Round[lsTimes[[i]]] <> " ns",
  FrameLabel → {"Radius (cm)", "Fusion Power (TW/g)"},
  DisplayFunction → Identity], {i, 1, Length[lsData[plotID3]}}];
```

Array of plots

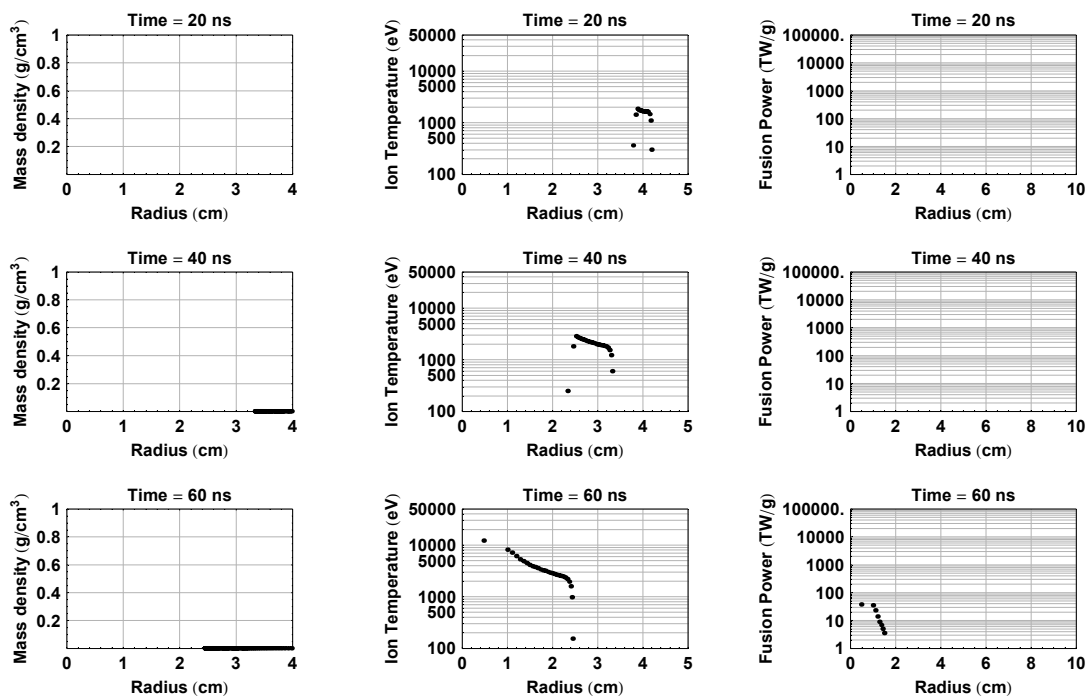
```
tbArray = Transpose[{tb1, tb2, tb3}];

Table[Show[GraphicsArray[tbArray[[{3 i + 1, 3 i + 2, 3 i + 3}]]],
  DisplayFunction → $DisplayFunction], {i, 0, 2}];
```

JFS

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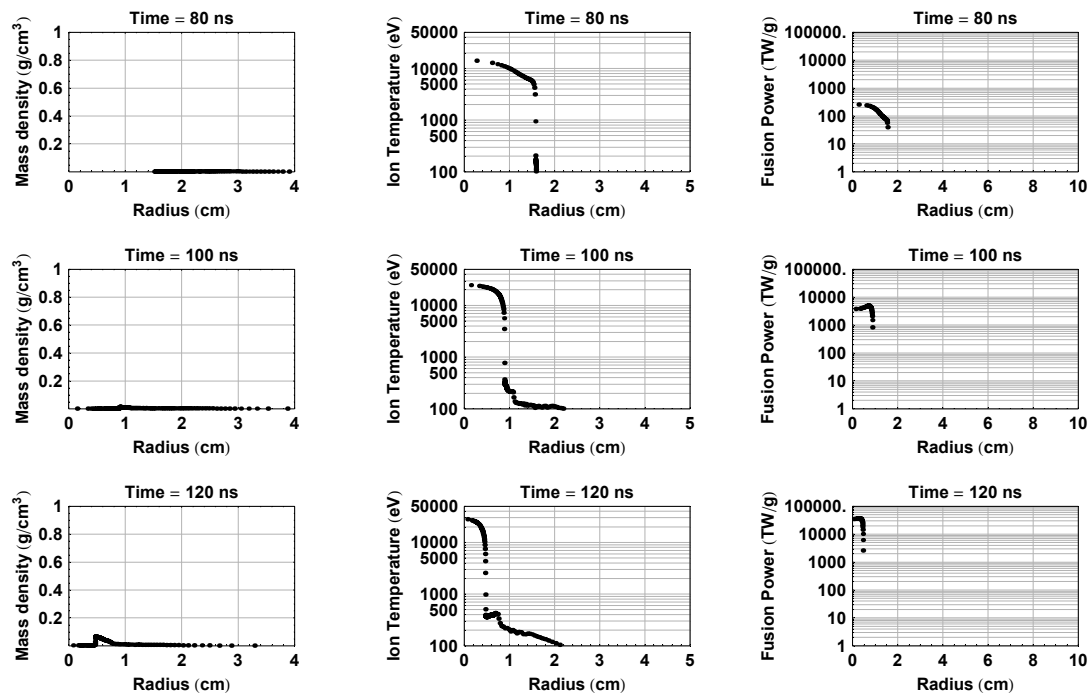
17:04:01



JFS

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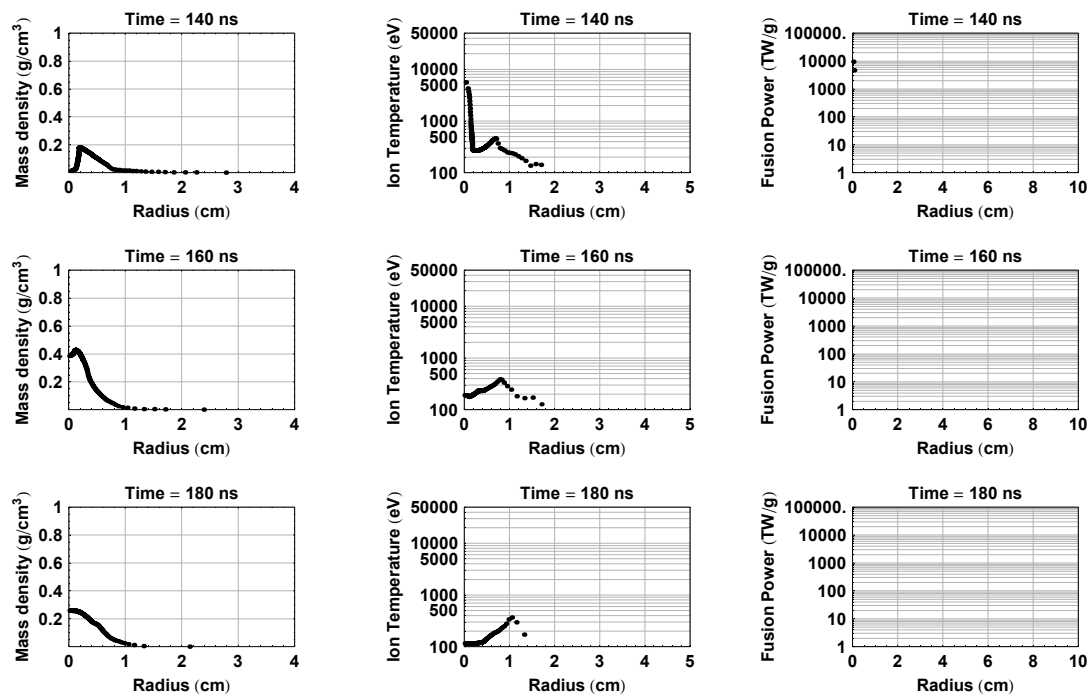
17:04:01



JFS

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17:04:02

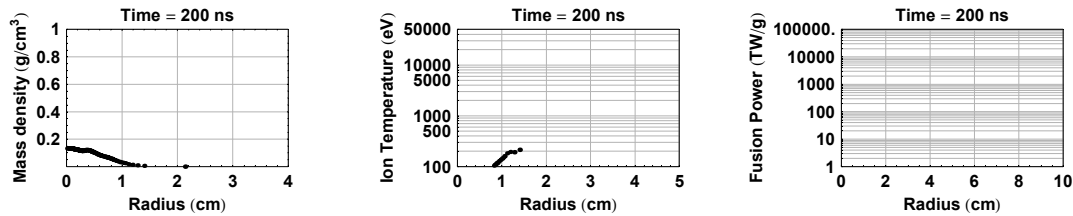


JFS

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17:04:02

```
Show[GraphicsArray[tbArray[[10]]], DisplayFunction -> $DisplayFunction];
```



☐ Undo options

JFS

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17:04:02

BUCKY case 030313.02: Thio, Current Trends (1999) 149 zones ($v_{jet}=125$ km/s)

■ Setup

```
caseID = "030313_02";

params = Graphics[Text["Case " <> caseID <> "\n
mt=4.38 mg; Δt=5.0 cm\n
mj=0.2 g; Δj=2.4 cm\n
mb=2.0 g; Δb=22.1 cm\n
vj=125 km/s; vb=125 km/s", {0, 0}, {0, 0}]];
```

■ 0-2 μs

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 2000;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";

thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

JFS

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17:04:02

Plots

$r < 0.3$ m

```

SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.3}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.07}, {0.95 * 10-3 tEnd, 0.17}], params],
  DisplayFunction → Identity];

SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
  Rectangle[{0.75 * 10-3 tEnd, 0.07}, {0.95 * 10-3 tEnd, 0.17}], params], DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt]];
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj]];
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz]];

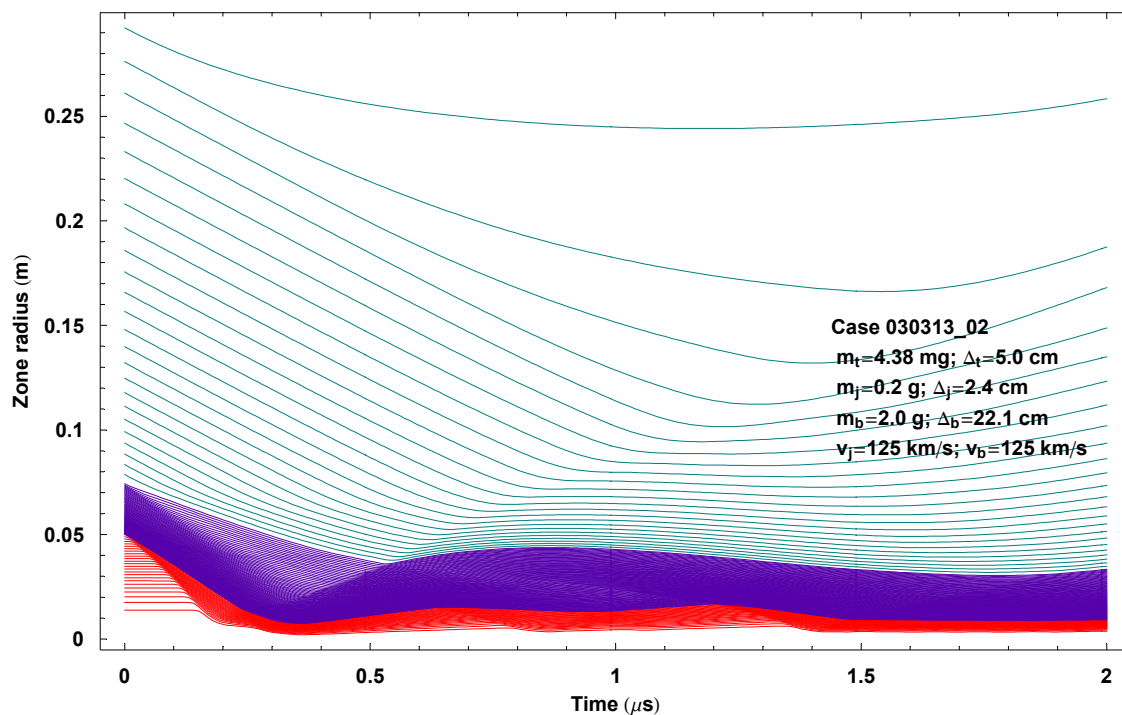
```

JFS

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



JFS

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17:04:02

• $r < 0.05$ m

```
SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.035}, {0.95 * 10-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];

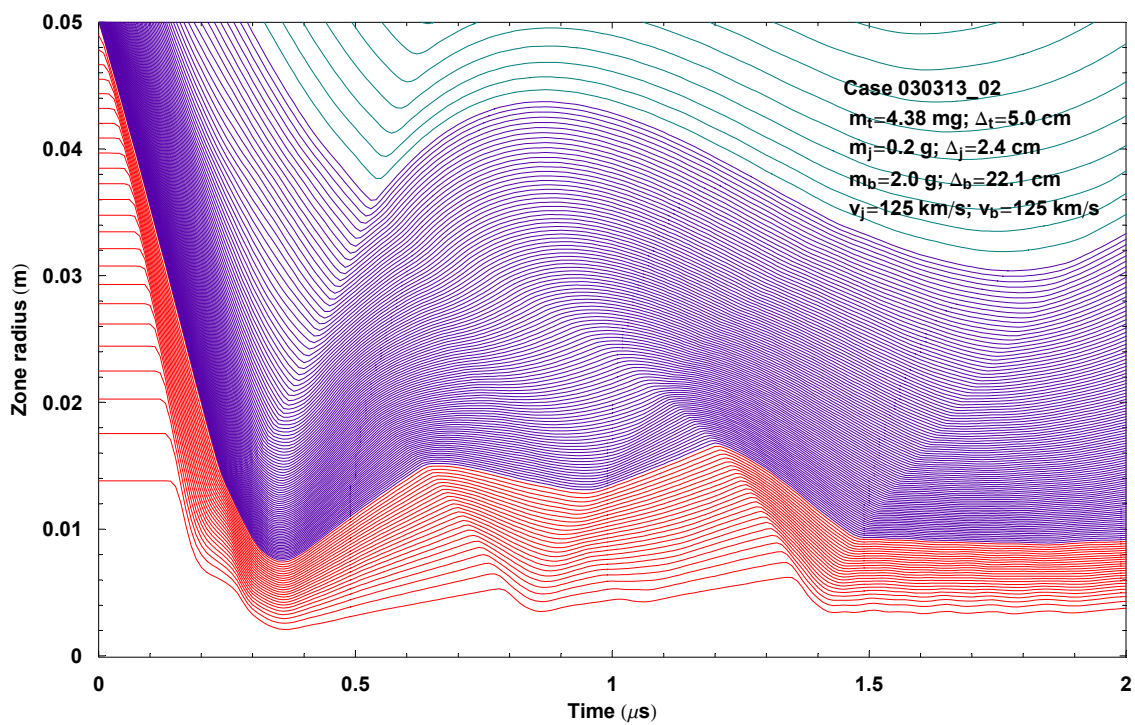
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt});
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj});
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz});
```

JFS

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



JFS

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• $r < 0.05$ m

```

tEndOrig = tEnd;
tEnd = 400;

SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.035}, {0.95 * 10-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt};
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj};
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz};

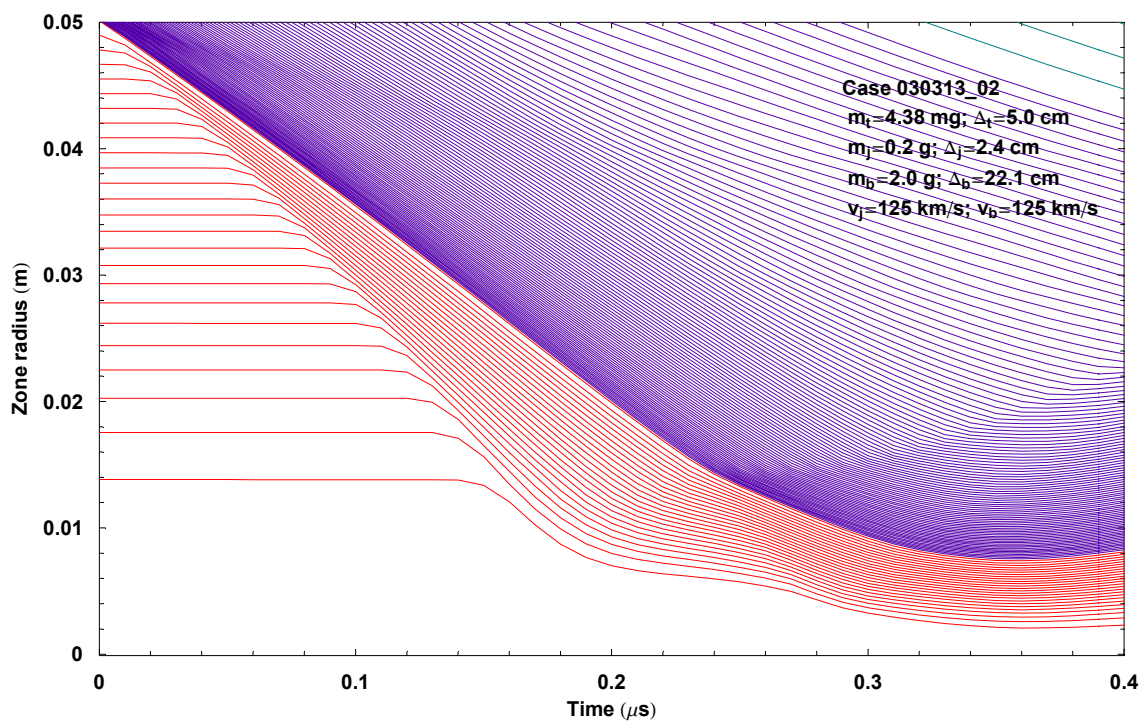
```

JFS

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



```
tEnd = tEndOrig;
```

JFS

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• $r < 0.02$ m

```
SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.02}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.7 * 10-3 tEnd, 0.014}, {0.9 * 10-3 tEnd, 0.018}, params],
  DisplayFunction → Identity];

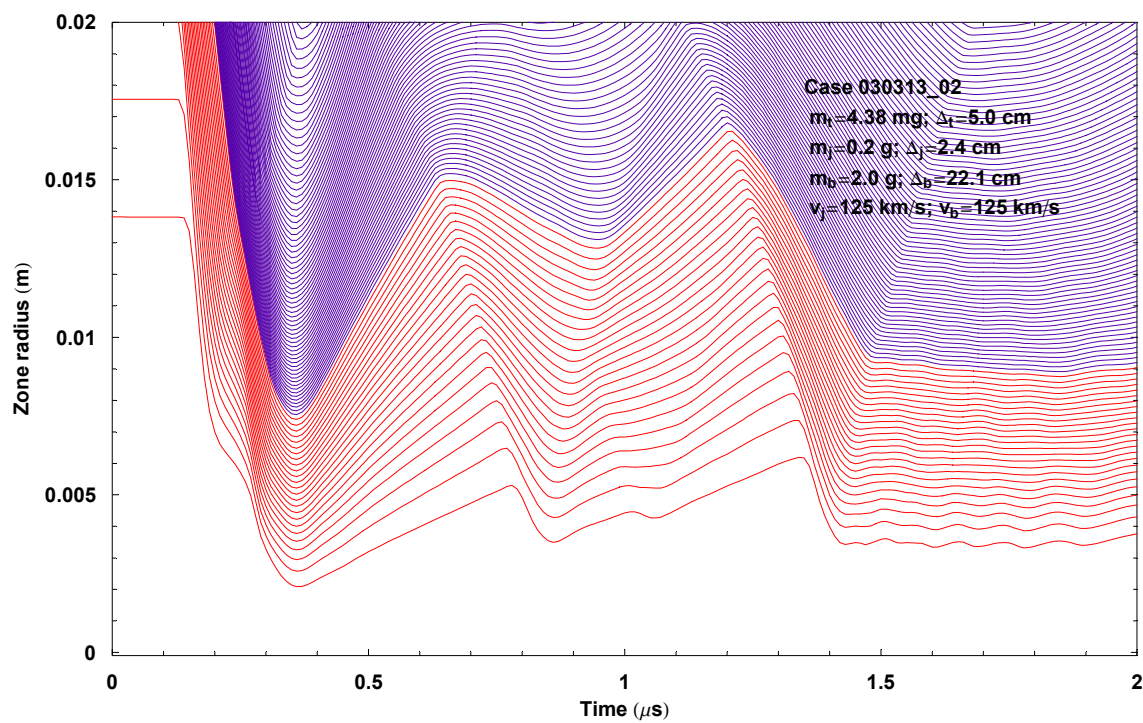
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}};
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}};
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}};
```

JFS

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



☐ Undo options

JFS

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■ Density vs time in each region (PLOT.regn.dens)

□ Set up problem and import data

□ Plot

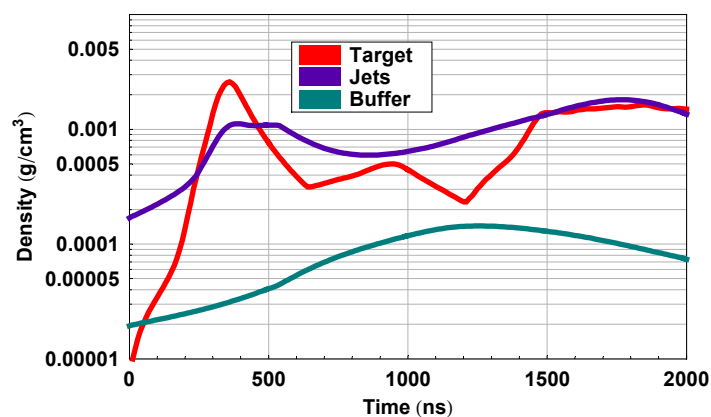
```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm3)"}];
plotrange = PlotRange → {{-10-3, tEnd}, {10-5.001, 10-2}};
p1ls = Table[LogListPlot[lsData[plotID][[All, i]], plotrange, PlotJoined → True,
  PlotStyle → lineStyle[[i, All]], DisplayFunction → Identity], {i, 1, 3}];
```

JFS

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```
ShowLegend[Show[p1ls],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {-0.2, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



□ Undo options

■ Ion temperature vs time in each region (PLOT.regn.temp)

□ Set up problem and import data

□ Plot

```
plotrange = PlotRange → {{-10-3, tEnd}, {0.999, 103}};
```

JFS

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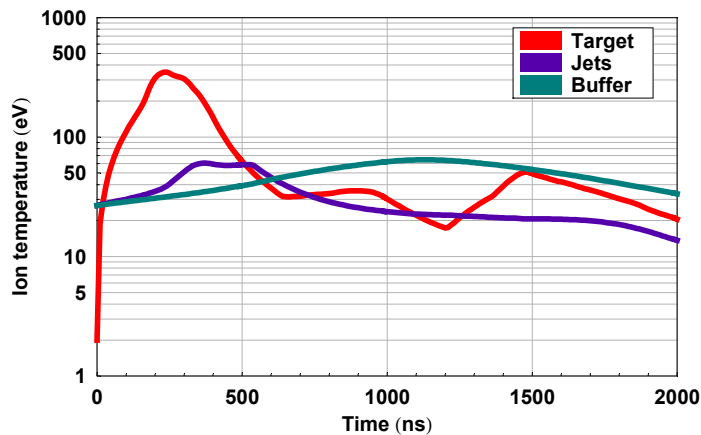
17:04:02

```

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[p11s],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



☐ Undo options

☒ Kinetic energy vs time in each region (PLOT.regn.KEs)

☐ Set up problem and import data

JFS

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17:04:02

☐ Plot

```

plotrange = PlotRange → {{-10-3, tEnd}, {10-5.0001, 100}};

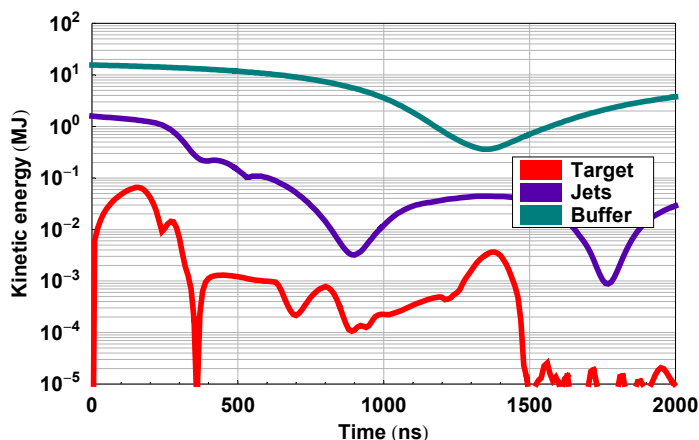
gridlinesY = Map[#, {GrayLevel[0.7]}] &, Flatten[Table[Log[10, j * 10i], {i, -5, 2}, {j, 1, 9}]]];

frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];

p11s = Table[LogListPlot[lsData[plotID][All, i], plotrange,
  PlotJoined → True, PlotStyle → lineStyle[[i]], DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[p11s, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
  FrameTicks → {Automatic, frameticksY, None, None}],
  {{{lineStyle[[1, 1], "Target"}, {lineStyle[[2, 1], "Jets"}, {lineStyle[[3, 1], "Buffer"}},
  LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];

```



JFS

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□ Undo options

■ Ion temperature vs radius (PLOT.tempn)

■ Mass density vs radius (PLOT.rho)

■ Fluid velocity vs radius (PLOT.vel)

■ Fusion power vs radius (BPlot variables 30 vs 1)

★ *Note: no fusion power was produced for this case.*

■ Combined plots: mass density, ion temperature, and *fluid velocity* vs radius

□ Setup

□ Table of plots

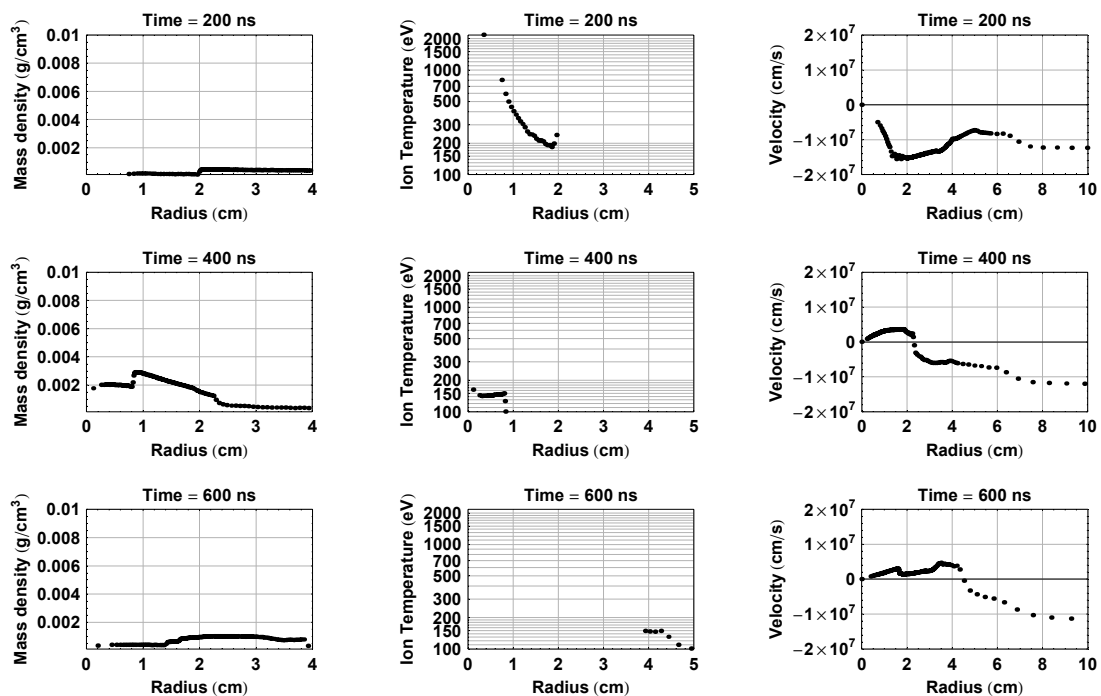
□ Array of plots

```
tbArray = Transpose[{tb1, tb2, tb3}];
Table[Show[GraphicsArray[tbArray[{{3 i + 1, 3 i + 2, 3 i + 3}}]],
  DisplayFunction -> $DisplayFunction], {i, 0, 2}];
```

JFS

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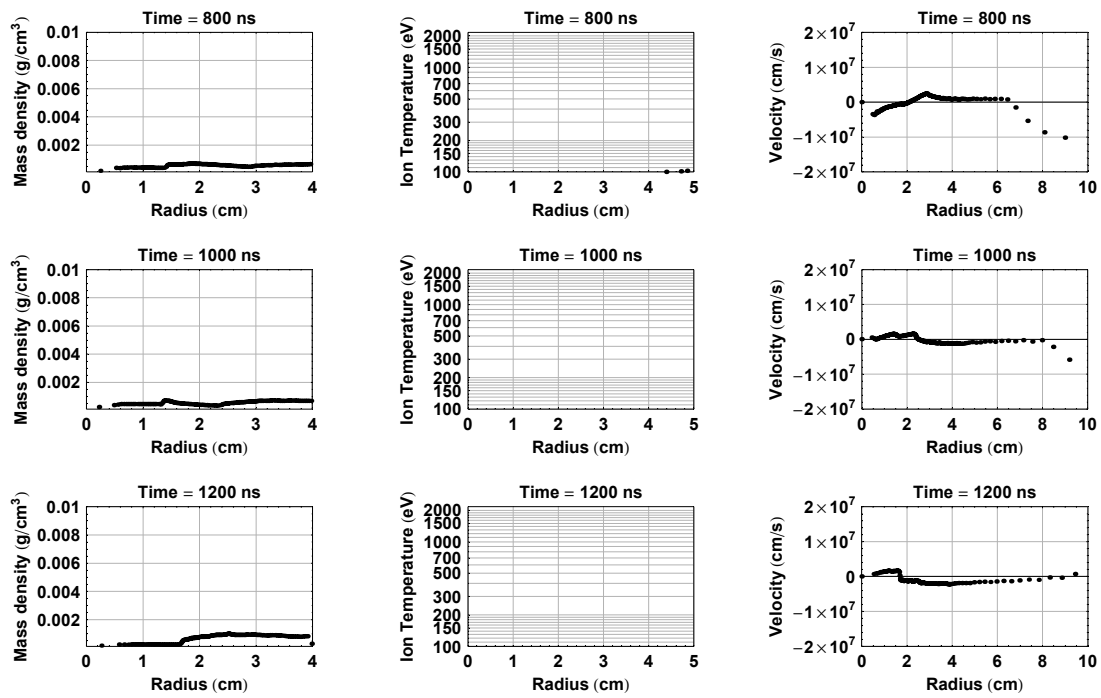
17:04:02



JFS

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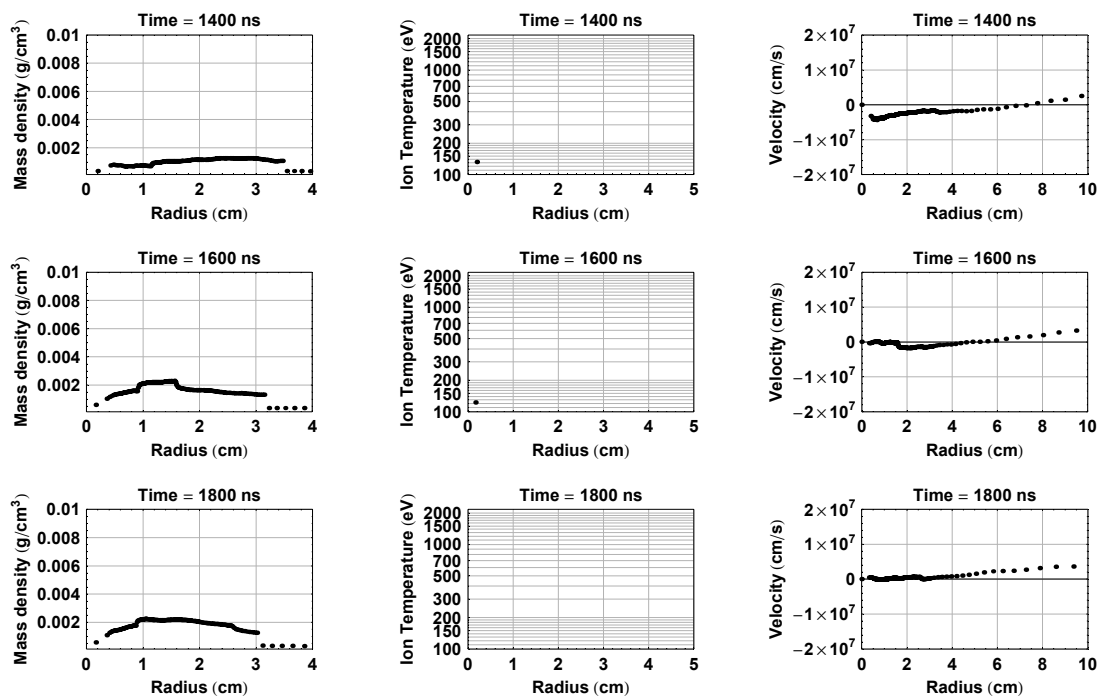
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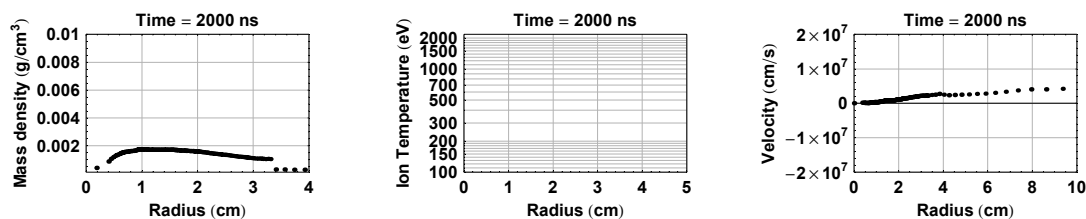


JFS

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```
Show[GraphicsArray[tbArray[[10]]], DisplayFunction -> $DisplayFunction];
```



□ Undo options

JFS

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BUCKY case 030313.04 ($v_{jet}=250$ km/s)

★ *Note: the full set of commands to generate the plots is shown in the section "BUCKY case 030313.02" above. In this section only the plots and the outline of the structure appear.*

■ Setup

```
caseID = "030313_04";

params = Graphics[Text["Case " <> caseID <> "\n
mt=4.38 mg; Δt=5.0 cm\n
mj=0.2 g; Δj=2.4 cm\n
mb=2.0 g; Δb=22.1 cm\n
Tej=Teb=106 eV\n
vj=250 km/s; vb=250 km/s", {0, 0}, {0, 0}]];
```

■ 0-500 ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 500;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";

thickness = Thickness[0.0002];
```

□ Read data

JFS

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- Number of time steps
- Number and list of radial zones in file
- Convert data from {s, cm} to {μs, m}
- Plots

- $r < 0.3$ m

```

SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.3}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.07}, {0.95 * 10-3 tEnd, 0.17}, params],
  DisplayFunction → Identity];

SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
  Rectangle[{0.75 * 10-3 tEnd, 0.07}, {0.95 * 10-3 tEnd, 0.17}, params], DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}};
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}};
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}};

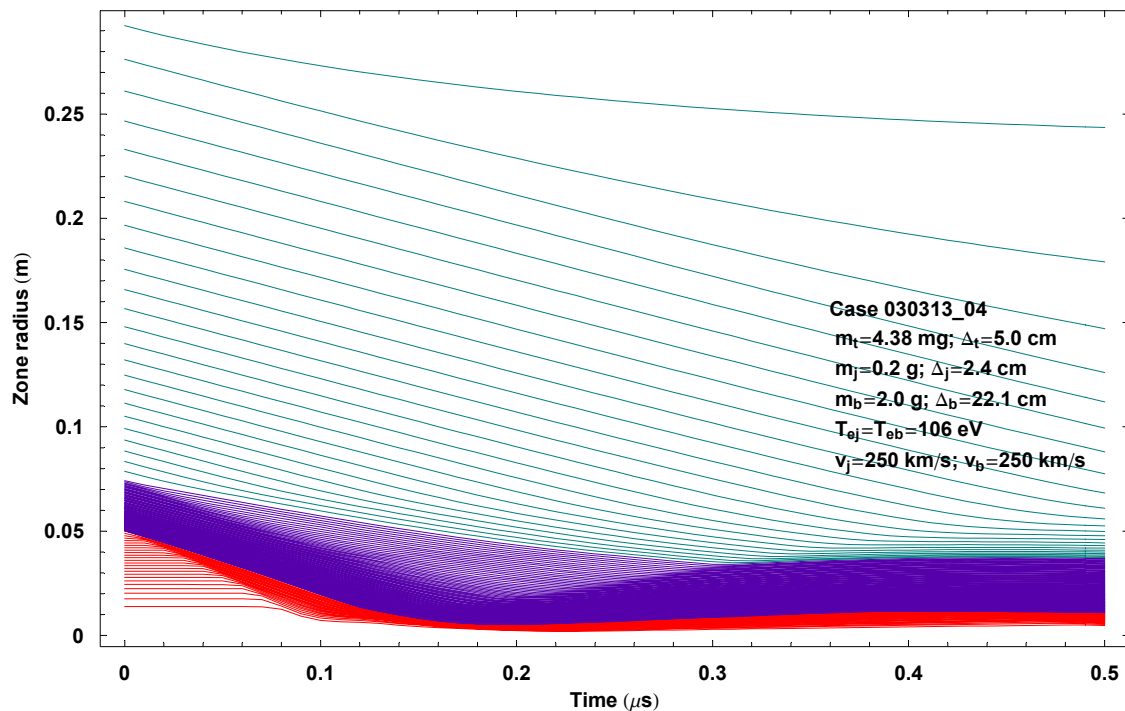
```

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



JFS

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• $r < 0.05$ m

```
SetOptions[ListPlot, PlotRange → {{-10-6, 10-3 tEnd}, {-10-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10-3 tEnd, 0.035}, {0.95 * 10-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];

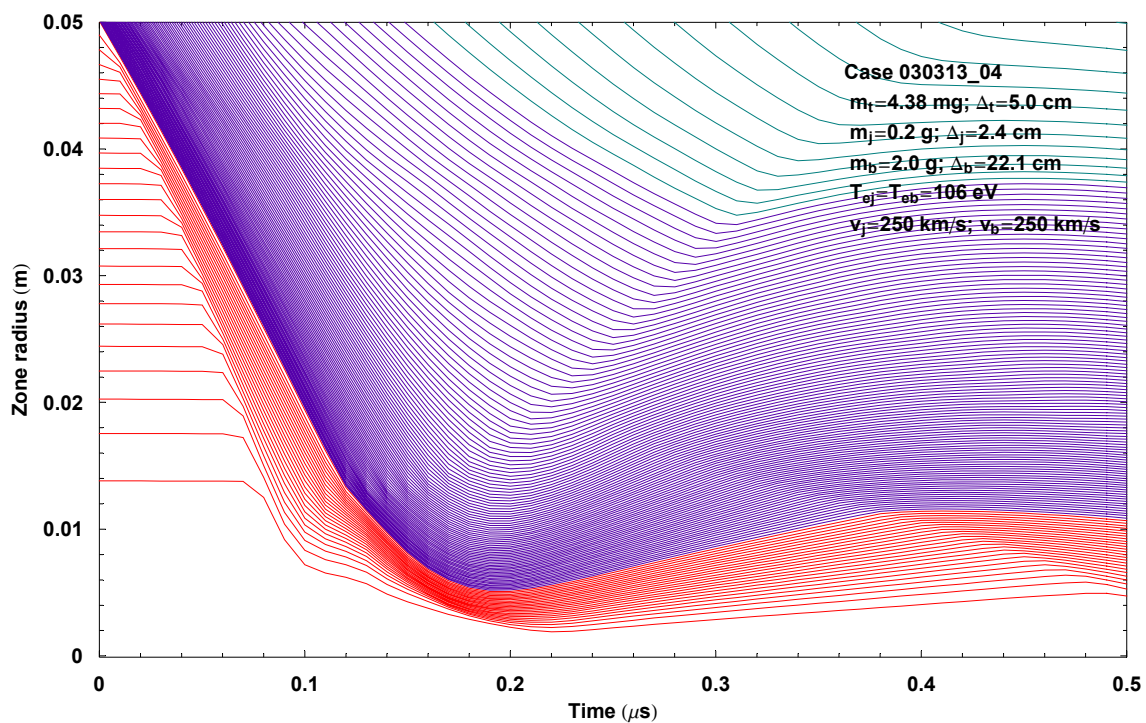
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, Nt}};
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, Nt + 1, Nt + Nj}};
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, Nt + Nj + 1, Nz}};
```

JFS

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```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



• $r < 0.02$ m

JFS

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□ Undo options

■ Ion temperature vs radius (PLOT.tempn)

□ Set up problem and import data

□ Plots

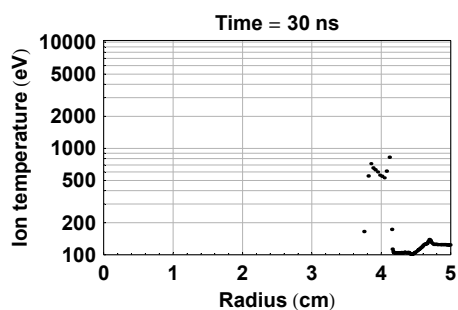
Partition length

```
partitionL = 2;
```

```
plotrangeTi = PlotRange → {{-10-3, 5}, {99.9, Max[lsData[plotID][[All, All, 2]]}};
```

```
tbData = Table[LogListPlot[lsData[plotID][[i]], plotrangeTi, PlotStyle → PointSize[0.01],  
  PlotLabel → "Time = " <> ToString[Round[lsTimes[[i]]] <> " ns", DisplayFunction → Identity],  
  {i, 1, Length[lsTimes]}];
```

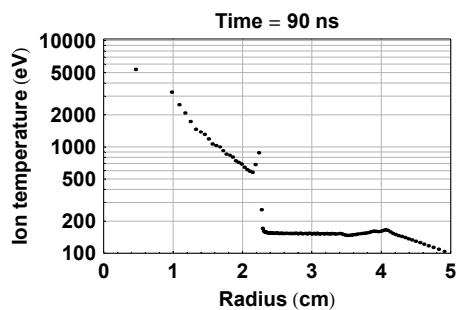
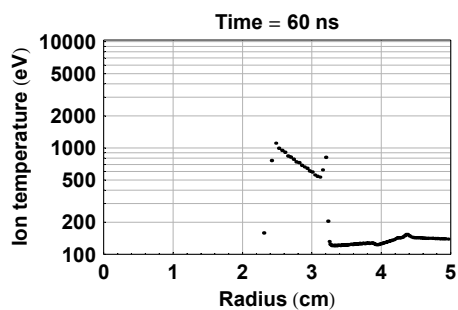
```
Map[Show[#, DisplayFunction → $DisplayFunction] &, tbData];
```



JFS

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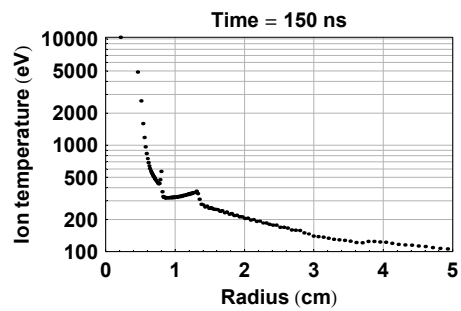
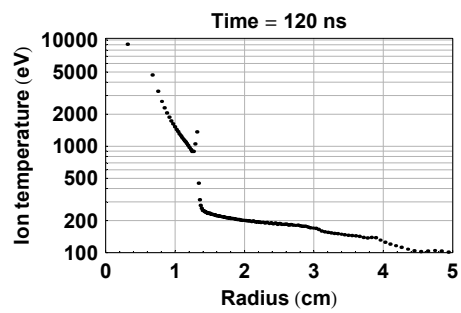
17:04:03



JFS

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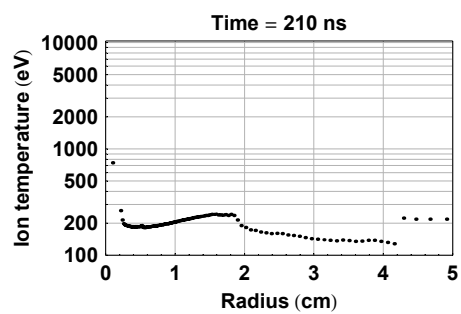
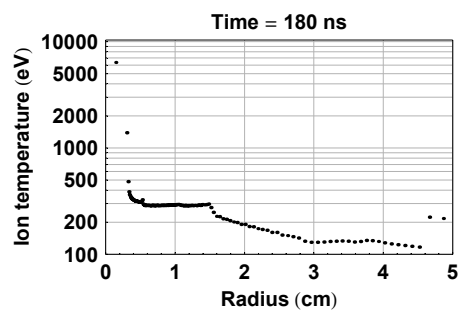
17:04:03



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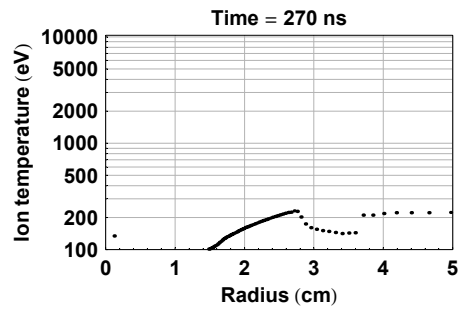
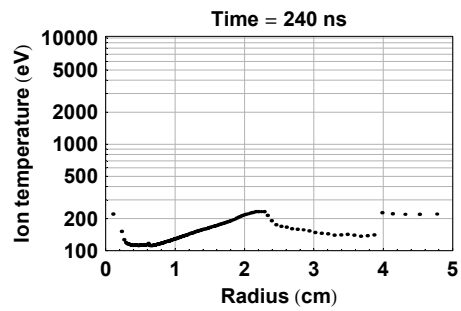
17:04:03



JFS

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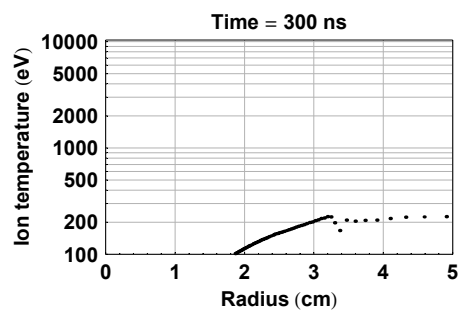
17:04:03



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☐ Undo options

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Neutronics

■ MCNP cases

■ Notes

Mohamed Sawan ran MCNP cases that roughly simulate neutron heating of the magnetic nozzle. The conductor and structure were approximated by 0.05 m of 316 stainless steel. The cross sections for the actual magnet materials do not differ too greatly, so this approximation should be sufficient for a zero-order analysis.

The list *lsNHeating* contains sublists with $\{\mathcal{E}_0, \mathcal{E}_n, \mathcal{E}_\gamma, \mathcal{E}_{\text{tot}}\}$, all in MeV, defined as

$\mathcal{E}_0 \equiv$ Initial neutron energy
 $\mathcal{E}_n \equiv$ Resulting neutron energy in magnet and structure
 $\mathcal{E}_\gamma \equiv$ Resulting gamma energy in magnet and structure
 $\mathcal{E}_{\text{tot}} \equiv$ Resulting total energy in magnet and structure

■ Data

```
lsNHeating = {{10-6, 0.099, 5.09, 5.18}, {10-5, 0.063, 3.18, 3.24},
  {10-4, 0.027, 2.53, 2.56}, {10-3, 0.071, 2.69, 2.76}, {10-2, 0.029, 1.16, 1.19},
  {0.030, 0.018, 0.47, 0.49}, {0.070, 0.021, 0.219, 0.240}, {0.1, 0.033, 0.225, 0.257},
  {0.3, 0.039, 0.077, 0.117}, {0.7, 0.077, 0.061, 0.138}, {0.8, 0.102, 0.063, 0.165},
  {0.93, 0.090, 0.122, 0.210}, {1, 0.099, 0.175, 0.274}, {2, 0.154, 0.560, 0.712},
  {7, 0.640, 2.53, 3.18}, {10, 0.76, 3.66, 4.42}, {14.1, 1.25, 3.31, 4.56}} // N;
TableForm[lsNHeating, TableHeadings → {None, { $\mathcal{E}_0$ ,  $\mathcal{E}_n$ ,  $\mathcal{E}_\gamma$ ,  $\mathcal{E}_{\text{tot}}$ }}
```

JFS

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□ Check:

```
lsNHeating[[All, 2]] + lsNHeating[[All, 3]]
  lsNHeating[[All, 4]]
{1.00174, 1.00093, 0.998828, 1.00036, 0.99916, 0.995918,
  1., 1.00389, 0.991453, 1., 1., 1.00952, 1., 1.00281, 0.996855, 1., 1.}
```

■ Resulting energy vs n energy

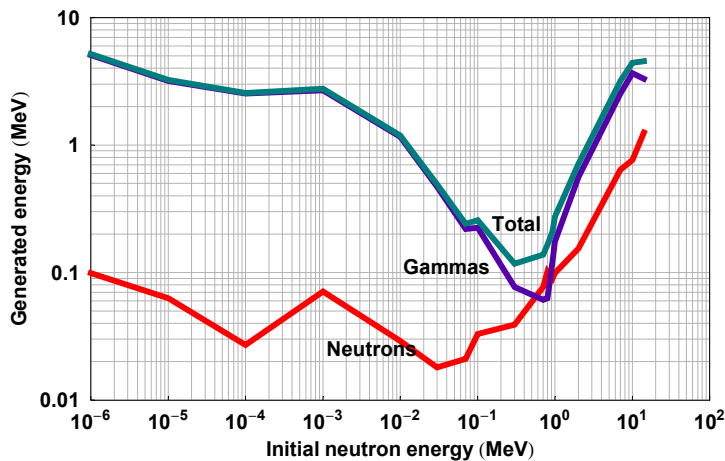
```
gridlinesX = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10i], {i, -6, 2}, {j, 1, 9}]]];
gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10i], {i, -3, 2}, {j, 1, 9}]]];
frameticksX = Table[{i, "\!\(10\^"<>ToString[i]<>"\)"}], {i, -6, 2}];
frameticksY = Table[{i, ToString[If[i < 0, N[10i], 10i]]}, {i, -3, 2}];
lllsPlots = Table[LogLogListPlot[Transpose[{lsNHeating[[All, 1]], lsNHeating[[All, i]]}],
  PlotRange → All, PlotStyle → lineStyle[[i - 1]],
  FrameLabel → {"Initial neutron energy (MeV)", "Generated energy (MeV)"},
  DisplayFunction → Identity], {i, 2, 4}];
text = {Graphics[Text["Neutrons", {-3.0, -1.6}, {-1, 0}]],
  Graphics[Text["Gammas", {-0.8, -0.85}, {1, 1}]],
  Graphics[Text["Total", {-0.5, -0.62}, {0, 0}]]];
```

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```
Show[lllsPlots, text, PlotRange -> {{-6.001, 2}, {-2.001, 1}},
  GridLines -> {gridlinesX, gridlinesY},
  FrameTicks -> {frameticksX, frameticksY, None, None}, DisplayFunction -> $DisplayFunction];
```



■ Energy multiplication vs n energy

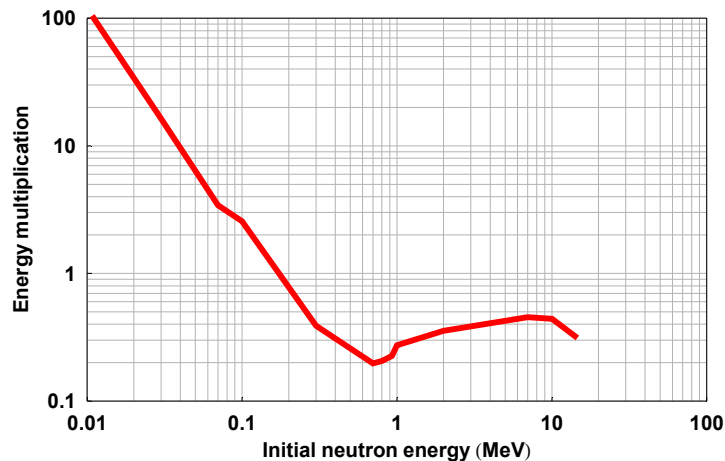
```
lllsNMult = LogLogListPlot[Transpose[{lsNHeating[All, 1],  $\frac{lsNHeating[All, 4]}{lsNHeating[All, 1]}$ }],
  PlotRange -> {{0.999 \times 10^{-2}, 100}, {0.0999, 10^2}}, PlotStyle -> lineStyle[[1], FrameLabel ->
  {"Initial neutron energy (MeV)", "Energy multiplication"}, DisplayFunction -> Identity];
frameticksX = Table[{i, ToString[If[i < 0, N[10^i], 10^i]]}, {i, -3, 2}];
```

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```
Show[lllsNMult, GridLines -> {gridlinesX, gridlinesY},
  FrameTicks -> {frameticksX, frameticksY, None, None}, DisplayFunction -> $DisplayFunction];
```



■ Energy absorbed by nozzle vs n energy

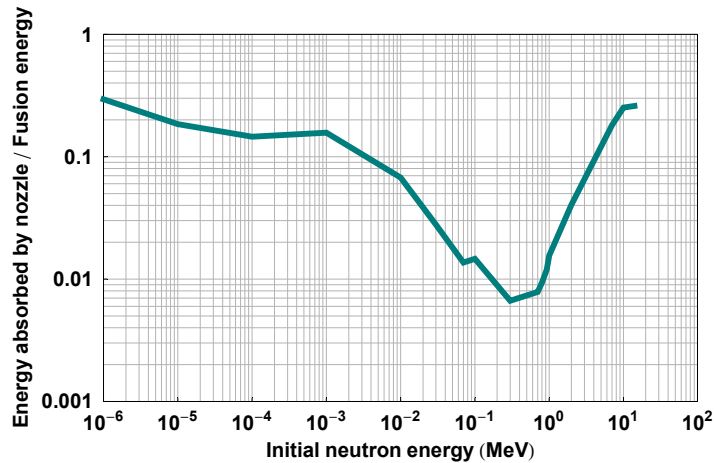
```
lllsNPfus = LogLogListPlot[Transpose[{lsNHeating[All, 1],  $\frac{lsNHeating[All, 4]}{17.58}$ }],
  PlotRange -> {{0.999 \times 10^{-6}, 100}, {0.0009999, 1}}, PlotStyle -> lineStyle[[3],
  GridLines -> {gridlinesX, gridlinesY}, FrameTicks -> {frameticksX, frameticksY, None, None},
  FrameLabel -> {"Initial neutron energy (MeV)", "Energy absorbed by nozzle / Fusion energy"},
  DisplayFunction -> Identity];
frameticksX = Table[{i, "\[10^" < ToString[i] < "\]"}, {i, -6, 2}];
```

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```
Show[lllsNPfus, GridLines -> {gridlinesX, gridlinesY},
FrameTicks -> {frameticksX, frameticksY, None, None}, DisplayFunction -> $DisplayFunction];
```



□ Function

```
lsNPfus = Transpose[{Log[10, lsNHeating[All, 1]],  $\frac{\text{lsNHeating[All, 4]}}{17.58}$ ]}];
nHt = Interpolation[lsNPfus];
Map[FindRoot[nHt[Log[10, En]] == 0.01, {En, #}] &, {0.2, 1}]
{{En -> 0.201552}, {En -> 0.854865}}
```

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```
FindMinimum[nHt[Log[10, En]], {En, 0.2, 0.5}]
{0.00620698, {En -> 0.459338}}
```

■ Calculation of neutron slowing down in deuterium

■ Setup

□ Read data

```
dataDirN = "c:\\nuclearphysics\\";
lsRT0 = Import[dataDirN <> "sigmaTotalVsE_ENDF_H-2f.dat"];
lsRT = Partition[Flatten[Drop[lsRT0, 2]], 2];
```

□ Function

```
logSigmaND = Interpolation[lsRT /. {x_, y_} -> {Log[10, x], y}]
InterpolatingFunction[{{-5., 8.17609}}, <>]
sigmaND[En_] := logSigmaND[Evaluate[Log[10, En]]]
```

□ Plots and check of function

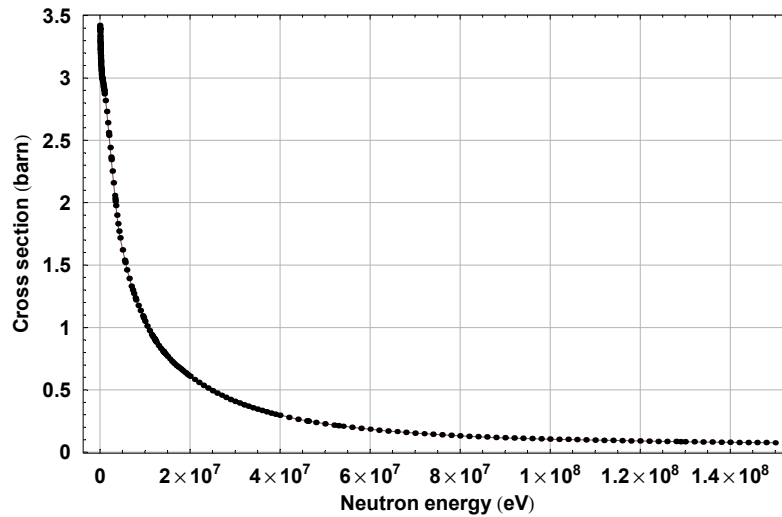
```
pls1 = ListPlot[lsRT, FrameLabel -> {"Neutron energy (eV)", "Cross section (barn)"},
PlotRange -> All, DisplayFunction -> Identity];
pls2 = Plot[sigmaND[En], {En, 100, 1.5*^8}, PlotRange -> All, DisplayFunction -> Identity];
```

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```
Show[pls2, pls1, PlotRange -> All, FrameLabel -> {"Neutron energy (eV)", "Cross section (barn)"},
  DisplayFunction -> $DisplayFunction];
```



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References

Thio 1999a: Y.C.F. Thio, E. Panarella, R.C. Kirkpatrick, C.E. Knapp, F. Wysocki, P. Parks, and G. Schmidt, "Magnetized Target Fusion in a Spheroidal Geometry with Standoff Drivers," Current Trends in International Fusion Research--Proceedings of the Second International Symposium (National Research Council of Canada, Ottawa, Canada, 1999).

Thio 1999b: Y.C.F. Thio, B. Freeze, R.C. Kirkpatrick, B. Landrum, H. Gerrish, and G.R. Schmidt, "High-Energy Space Propulsion Based on Magnetized Target Fusion," 35th AIAA/ASMA/SAE/ASEE Joint Propulsion Conference paper AIAA-99-2703 (1999).

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